3. FORCE OSCILLATIONS

Introduction-

When body oscillates due to restoring force but oscillations of body die out due to resistive force of the medium these oscillations are called are called damped oscillations. When we maintain the oscillations of the body external force must be required periodically, this force is called **periodic force**. This force maintains the oscillations so called driving force. For example swing machine.

Equation of force oscillation-

When a body is set into oscillations due to external periodic force, three forces are acts on the body.

1. Restoring force
2. Damping force
3. The external resistive force

$\boldsymbol{Restoring\ force \propto x}$ $\qquad f \propto x\ and\ f \propto \frac{dx}{dt}$

$\boldsymbol{Resistive\ force\ \propto velocity}$ $\qquad f = -kx\ and\ f = -R\frac{dx}{dT}$

The external periodic force “f_0 sinqt ,” Where f_0 is amplitude and q is angular frequency of the applied force. The total force acting on the body is,

$$F = -kx - R\frac{dx}{dt} + f_0 \sin qt$$

According to Newton’s 2nd laws,

F= ma

$$F = m\frac{d^2x}{dt^2}$$

Compare above to equation, we get second order, linear and homogeneous differential equation in x.

$$m\frac{d^2x}{dt^2} + R\frac{dx}{dt} + kx = f_0 \sin qt$$

This is differential equation of forced oscillations. This is second order, linear and inhomogeneous differential equation in x. It has two parts,

1. Complimentary function
2. Particular integral

Complimentary function is the solution of equation

$$m\frac{d^2x}{dt^2} + R\frac{dx}{dt} + kx = 0$$

The oscillatory solution of the above equation is,

$$x = ae^{-\frac{Rt}{2m}}\sin(pt + \theta)$$

P is the angular frequency of damped oscillations. The particular integral of the above equation is, $x = Asin(qt - ϕ)$

Where, A and ϕ are constants. Thus, the general solution of equation is,

$x = ae^{-\frac{Rt}{2m}}\sin(pt + \theta)$ + A sin (qt- ϕ)

The initial stage, in which the two types of motion are both, prominent is called transient.

$$A = \frac{f_0}{\sqrt{(k - mq^2)^2 + R^2q^2}}$$

$$tanϕ = \frac{Rq}{k - mq^2}$$

Characteristics of forced Oscillations:

1. **Amplitude-**

The amplitude of forced oscillations is given by,

$$A = \frac{f_0}{\sqrt{(k - mq^2)^2 + R^2q^2}}$$

We have, $k = m\omega^2$

ω = natural angular frequency of the body

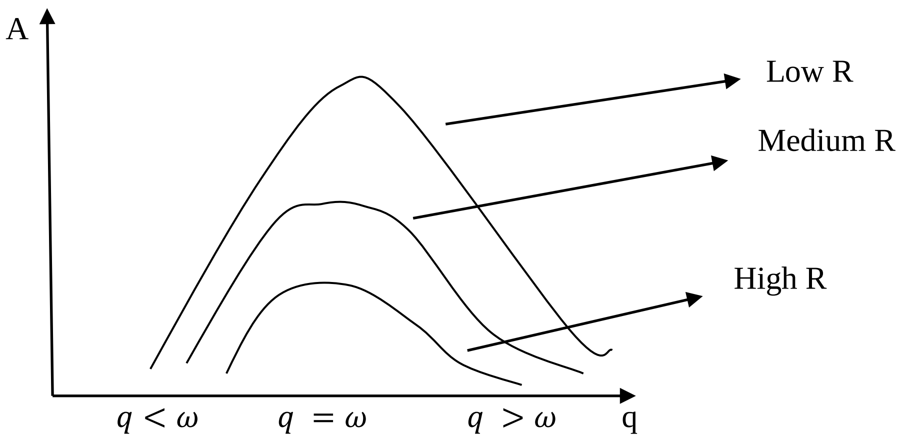
$$A = \frac{f_0}{\sqrt{(m\omega^2 - mq^2)^2 + R^2q^2}}$$

$$A = \frac{f_0}{m\sqrt{(\omega^2 - q^2)^2 + \frac{R^2q^2}{m^2}}}$$

Thus, amplitude is independent of time, but depends on R and$(\omega^2 - q^2)$.

Amplitude increases as$(\omega^2 - q^2)$ decreases and R decreases.

The variation of the amplitude A with angular frequency of applied force as shown in figure



Variation of amplitude with frequency

2. Phase-

The phase of oscillation is given by

$$tan\phi = \frac{Rq}{k - mq^2}$$

$$tan\phi = \frac{Rq}{m\omega^2 - mq^2}$$

$$tan\phi = \frac{Rq}{m(\omega^2 - q^2)}$$

The phase depends on$(\omega^2 - q^2)$ and R

Condition	tan ϕ	Angle ϕ
$q < \omega$	Positive	$0 < \phi < \frac{\pi}{2}$
$q = \omega$	Infinity	$\frac{\pi}{2}$
$q > \omega$	Negative	$\frac{\pi}{2} < \phi < \pi$

3. Frequency-

The angular frequency of forced oscillations is same as that of forcing frequency q , thus frequency of forced oscillations is $\frac{q}{2\pi}$.

The mechanical resonance has two types

1. Amplitude resonance
2. Velocity resonance

1. **Amplitude resonance-**

The amplitude of forced oscillation is,

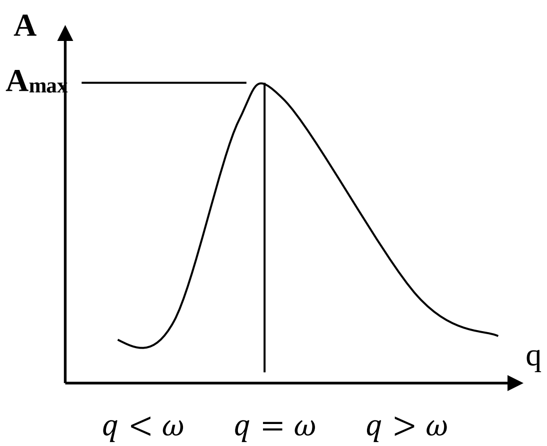
$$A = \frac{f_0}{m\sqrt{(\omega^2 - q^2)^2 + \frac{R^2 q^2}{m^2}}}$$

The amplitude of oscillator is Maximum when denominator of above equation is Minimum. The denominator is depends on driving frequency q.

$$q = \sqrt{\omega^2 - \frac{R^2}{2M^2}}$$

Angular frequency of forced oscillations at which amplitude is maximum, when damping is very small, we can neglect the term $\frac{R}{2m}$ and we can get $q = \omega$,

Thus, for low damping the amplitude of oscillation is Maximum, the **angular frequency of applied force is equal to natural frequency called amplitude resonance.**



When $q = \omega$ amplitude is maximum and is given by,

$$A_{max} = \frac{f_0}{\omega R}$$

A_{max} is inversely proportional to damping coefficient R.

R=0, $A_{max} = \infty$.

2. **Velocity resonance-**

The displacement of forced oscillation is x=A sin (qt-ф)

Velocity V= $\frac{dx}{dt}$

V = AqCos (qt-ϕ)

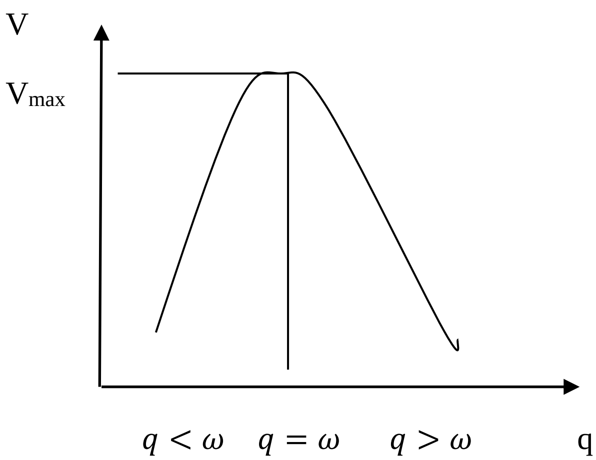
V =V_0 cos(qt-ϕ)

V_0=Aq called velocity amplitude.

$V_0 = \frac{f_0}{m\sqrt{(\omega^2 - q^2)^2 + \frac{R^2 q^2}{m^2}}}$

The velocity of amplitude is depends upon denominator, The velocity amplitude is maximum denominator is minimum.

$q = \omega$ The velocity amplitude is maximum, the driving angular frequency is equal to natural frequency is called velocity resonance.



$q = \omega$, $V_{max} = \frac{f_0}{R}$

At velocity resonance,

tanϕ=∞, ϕ =$\frac{\pi}{2}$

i.e. at resonance the displacement and applied force have phase difference $\frac{\pi}{2}$ or 90^0

Energy equation of forced oscillations-

$m\frac{d^2x}{dt^2} + R\frac{dx}{dt} + kx = f_0 \sin qt$ Is the equation of forced oscillation used for calculating the energy of forced oscillations, Total energy =K. E +P. E

Total energy is the average rate of absorption of energy due to oscillating body and the average rate of dissipation of energy due to external damping force, these values are equal in magnitude and opposite in direction, so average energy over a cycle of oscillation is constant. i. e. $\vec{E} = Constant.$

Quality factor of forced oscillation-

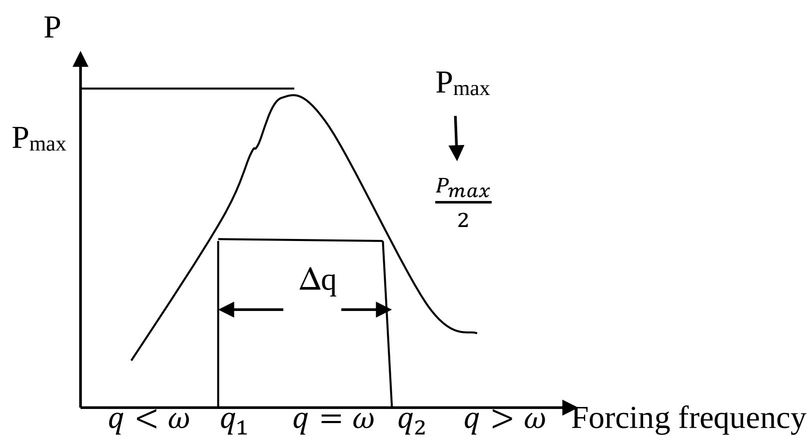
The quality factor is an efficiency of oscillator.

$$Q = 2\pi \frac{Energy\ stored}{Dissipation\ of\ energy\ in\ one\ oscillation}$$

$$Q = \frac{m\omega}{R}$$

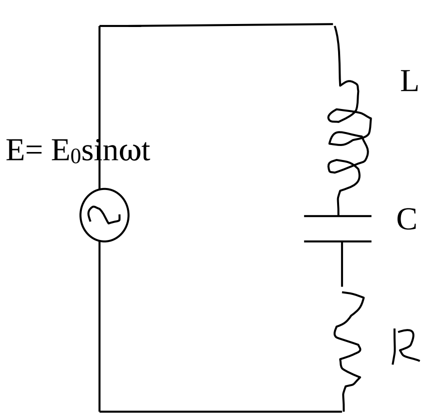
Power and Bandwidth of forced oscillation-

The Bandwidth Δq is defined as the difference in angular frequency of applied force for which power falls from $\vec{P}_{max}$ to $\frac{\vec{P}_{max}}{2}$ and $\vec{P}_{max}$= $\frac{f_0^2}{2R}$



Bandwidth of forced oscillation is, Δq = $\frac{R}{m}$ and Quality factor Q is,
$Q = \frac{m\omega}{R}$ so, Q = $\frac{\omega}{\Delta q}$

LCR series circuit-



Electrical resonance-

The current through LCR circuit is maximum when impedance is minimum.

Peak value of current is,

$$I_0 = \frac{E_0}{\sqrt{(\frac{1}{\omega' c} -^{L\omega'}) + R^2}}$$

Here, this equation is shows impedance of the circuit and it depends upon frequency of applied emf, impedance of the circuit will be minimum at particular angular frequency $\omega' = \omega$

$(\frac{1}{\omega' c} -^{L\omega'})\omega'$=0

$(\frac{1}{\omega' c} -^{L\omega'})$=0

$$\omega = \frac{1}{\sqrt{LC}}$$

This frequency is called resonant Angular frequency for which current is maximum.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

At resonance, tanΦ= $\frac{R}{\frac{1}{\omega C} - \omega L}$ =infinity At, Φ =$\frac{\pi}{2}$