Chapter 4

Wave Motion

Introduction-

The motion of an oscillatory disturbance through the medium is called wave motion. The wave motion is mode of transfer of energy. Wave over a stretched string, light waves, sound waves, and seismic waves are examples of waves. Due to vibration of particles and direction of propagation waves can be classified into two types,

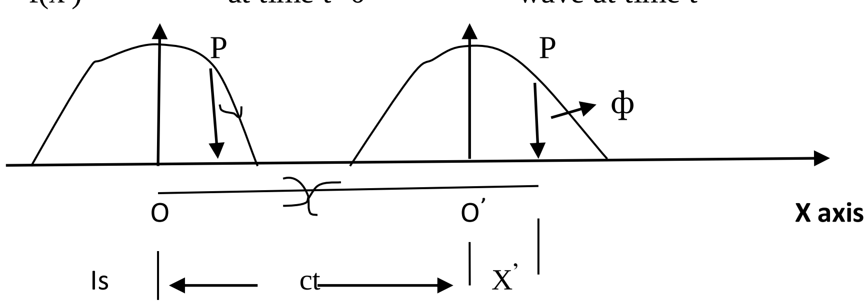
1. Transverse waves
2. Longitudinal waves

1. **Transverse waves-** The wave in which particles of the medium **vibrates perpendicular to the direction of propagation** of waves are called transverses waves, medium must have elasticity of shape. Only solid possesses elasticity of shape so transverses waves can travel through the solid only. E.g. light waves
3. **Longitudinal waves -** The wave in which particles of the medium **vibrates parallel to the direction of propagation** of waves is called Longitudinal waves, the medium through which the longitudinal waves travel must have elasticity of volume i.e. bulk modulus. The longitudinal waves can travel through solids, liquids and gases. The pulse passing through oscillating spring and sound waves are longitudinal waves.

Differential equation of wave motion-

Consider pulse travelling along positive x axis with speed c, during motion particles vibrate about their mean positions. Displacement of particle is ϕ. At time t waveform is,

$\Phi = f(x')$ at time t=0 wave at time t



Point P is at particular location on the phase og the pulse at time t = 0,

After time t the point P on the phase has shifted a distance ct in the positive x direction. Since origin shifted o to o'

$$x' = x - ct$$

$$\phi = f(x - ct)$$

If a wave is travelling along the negative direction of x-axis the equation of motion is,

$$\phi = f(x + ct)$$

To obtain differential equation of wave motion, consider above equation,

$$\phi = f(x - ct)$$

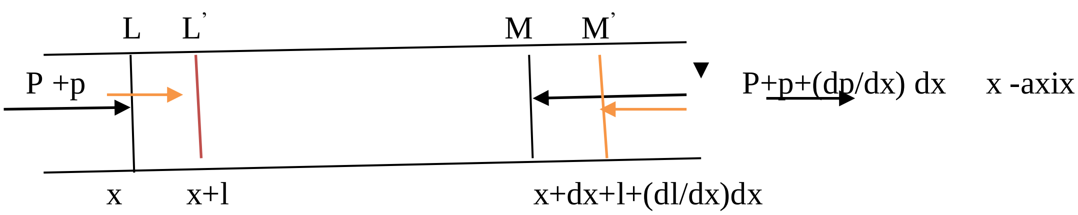
Double differentiation of above equation w.r.to x and t, we get

$\frac{d^2\phi}{dx^2} = \frac{1}{c^2}\frac{d^2\phi}{dt^2}$ This is equation of motion.

Differential equation of longitudinal waves-

Longitudinal disturbance is propagating through the fluid (gas or liquid) of density ρ.

Consider an element of liquid in horizontal tube as shown in figure,



The distance of planes LM is xand x+dx from origin.

Length of element LM = dx

Volume of element LM, V= Adx

Mass of the element LM , m= ρAdx

When no wave propagates through the medium, the element LM is equilibrium and pressures on planes L and M equal and opposite. If tube receives the disturbance from left, it undergoes small displacement. LM shifted to L'M' respectively.

Element undergoes volume strain due to stress because longitudinal waves travel must have elasticity of volume.

K=$\frac{volume\ stress}{volume\ strain}$

After calculations,

$$\frac{d^2l}{dt^2} = \frac{\rho}{K}\frac{d^2l}{dt^2}$$

General differential equation of wave motion is,

$\frac{d^2\phi}{dx^2} = \frac{1}{c^2}\frac{d^2\phi}{dt^2}$

Where c is velocity of wave, Compare above equations,

$$\frac{1}{c^2} = \frac{\rho}{K}$$

$$C = \sqrt{\frac{K}{\rho}}$$

C is velocity of longitudinal waves propagating through a fluid of density ρ, the velocity depends on bulk modulus K and density ρ.

Equation of motion of SM Progressive Longitudinal wave-

During propagation, equation of longitudinal wave is,

$$l = l_0 sink(x - ct)$$

But, $\omega = kc$

So, $l = l_0 \sin(kx - \omega t)$

$$\omega = \frac{2\pi}{T}$$

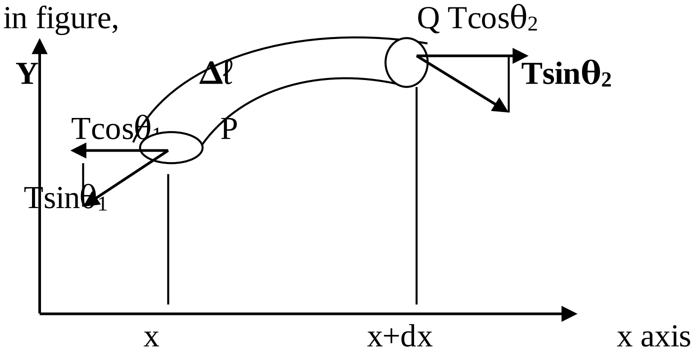
$$k = \frac{2\pi}{\lambda}$$

The equation of wave can also written as,

$$l = l_0 sin2\pi(\frac{x}{\lambda} - \frac{t}{T})$$

Differential equation of a transverse wave-

Consider a element of the stretched string in vibration mode. T is uniform tension and μ is mass per unit length. Wire undergoes small displacement along Y axis. Δℓ is length of the element in displaced condition. $\boldsymbol{\theta_1}$ **angle made by tension T with horrizontal axis at P** and $\boldsymbol{\theta_2}$ **angle made by tension T with horrizontal axis at Q**. T is tension along x and y axis as shown in figure,



The force along **x axis** is,

$$F_x = T\cos\theta_2 - T\cos\theta_1$$

$\boldsymbol{\theta_1}$ And $\boldsymbol{\theta_2}$ are very small, $\cos\theta_2 and\ \cos\theta_1$=1

$$F_x = 0$$

The force along y axis will be,

$$F_y = T\sin\theta_2 - T\sin\theta_1$$

If $\boldsymbol{\theta_1}$ And $\boldsymbol{\theta_2}$ are very small, $\sin\theta_1 = \tan\theta_1$ and $\sin\theta_2 = \tan\theta_2$

$$F_y = T\tan\theta_2 - T\tan\theta_1$$

$\tan\theta_1$ And $\tan\theta_2$ represent slopes of the string element at point P and Q respectively, therefore,

$$\tan\theta_1 = (\frac{dy}{dx})_P\ and\ \tan\theta_2 = (\frac{dy}{dx})_Q$$

Put in above equation, after calculation, we get

$$\frac{d^2y}{dx^2} = \frac{\mu}{T}\frac{d^2y}{dt^2}$$

This is differential equation of transverse wave on a string. General equation of wave motion is,

$$\frac{d^2\phi}{dx^2} = \frac{1}{c^2}\frac{d^2\phi}{dt^2}$$

C is velocity of the wave, Compare above equation, we get

$$c = \sqrt{\frac{T}{\mu}}$$

C is velocity of transverse wave on the string.

Equation of motion of a SM progressive transverse wave-

Displacement of transverse wave motion is,ϕ = y, i.e. y = f (x-ct).

During propagation of SHPT wave particle execute SH motion perpendicular to the direction of propagation. So the function is sin or cosine of (x- ct) ,

So equation is, y = $y_0$sin k (x- ct) this equation represent SHM of angular frequency $\omega = \frac{2\pi}{T}$

Using ω=kc ,

y =$y_0$sin (kx- ωt) along x axis

If we put,

$k = \frac{2\pi}{\lambda}$ And $\omega = \frac{2\pi}{T}$

$$y = asin2\pi(\frac{x}{\lambda} - \frac{t}{T})$$

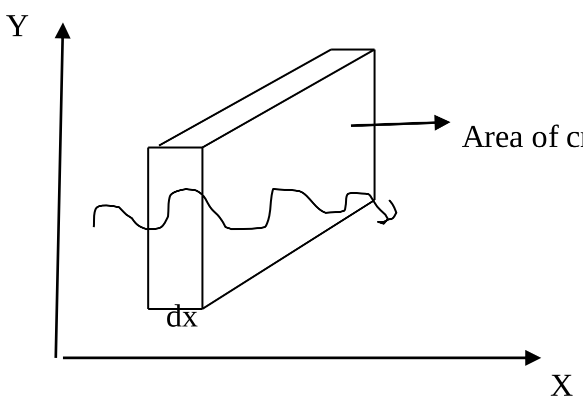
Energy of SM progressive waves-

In case of progressive wave motion, there is **no transfer of medium** in the direction of propagation, but there is **always transfer of energy** in the direction of propagation of wave. The energy is partly kinetic and partly potential. The K.E is due to velocity of vibrating particles and P.E is due to the displacements of particles from their mean position.

Consider a wave travelling along positive X-direction. The displacement of particle is y, and then equation of wave motion is,

y = a sin $(\omega t - kx)$

Let us consider a thin layer of the medium of thickness dx, density ρ and cross section area S.



$$\frac{dy}{dt} = a\omega \cos\,(\omega t - kx)$$

K.E of the layer, $K.E = \frac{1}{2} Mv^2$

P.E = $-\int_0^y F\,dy$ F=ma

$$E = \frac{1}{2}\rho(Sdx)a^2\omega^2$$

Energy per unit volume or energy density of the wave is,

$$E_v = 2\pi^2 v^2 \rho a^2$$

$$E_v \propto a^2$$

Energy density is directly proportional to the square of the amplitude of the wave.

Seismic wave-

The study of earthquakes and other movements of the earth's crust are called seismology. In seismology the study involves occurrence, propagations and detection of seismic waves. This wave produced from stored energy such as elastic strain, chemical energy or gravitational energy is released suddenly. The disturbance produced at any place inside the earth, send waves through the body of the earth. These waves are called seismic waves.

The place within the earth, where the disturbance starts is called **focus.** The place immediately above the focus on the surface of the earth is called **epicenter. These waves from sudden breaking of rock within the earth**

Types of seismic waves –There are two main types

1. **Body waves**- It travelling through the interior part of the earth surface, these waves arrive before the surface waves emitted by an earthquake. These waves are higher frequency than surface waves. Depending upon the mode of vibration, it has two types,
   1. Primary or pressure waves or P waves
   2. Secondary or shear or S waves
2. **Surface waves-**The seismic waves originate from the epicenter, called **surface waves**. The waves travel along the surface of the earth has two types
   1. Rayleigth or R waves
   2. Lateral waves or L-waves

Gravitational waves-

1. **Continuous gravitational waves**
2. **Compact Binary In spiral gravitational waves**
3. **Stochastic gravitational waves**
4. **Burst gravitational waves**