Total No. of Questions : 4]

SEAT No. :

**P 920**

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# [6054]-101
## S.Y.B.Sc. (Regular)
## MATHEMATICS
## MT - 231 : Calculus of Several Variables
## (2019 Pattern) (Credit system) (Semester - III) (Paper -I) (23111)

*Time : 2 Hours]* *[Max. Marks : 35*

***Instructions to the candidates:***

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*

***Q1)*** Attempt any FIVE of the following. **[5]**

a) Evaluate $f(3,2)$, if $f(x,y) = x\ ln\ (y^2 - x)$

b) show that

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

does not exist.

c) If $f(x,y) = 4 - x^2 - 2y^2$, find $f_x\ (1,1)$

d) Define wave equation.

e) Find the critical points of a function $f(x,y) = y^2 - x^2$.

f) Find $\int_0^5 f(x,y)\,dx$, if $f(x,y) = 12x^2y^3$.

g) Find the Jacobian of the transformation $x = 5u - v$, $y = u + 3v$.

***Q2)*** a) Attempt any ONE of the following. **[5]**

i) Define function of two variables, domain and range of function of two variables. Find the domain and range of $f(x,y) = \sqrt{9 - x^2 - y^2}$

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ii) Define two dimensional Laplace equation and harmonic functions. Show that the function $u(x,y) = e^x \sin y$ is a solution of laplace equation.

b) Attempt any one of the following. **[5]**

i) Determine the set of points at which the function $f(x,y,z) = \arcsin(x^2+y^2+z^2)$ is continuous.

ii) If $z = f(x,y) = x^2+3xy-y^2$ and $x$ changes from 2 to 2.05 and $y$ changes from 3 to 2.96, compare the values of increment $\Delta z$ and the differential dz.

***Q3)*** a) Attempt any one of the following. **[5]**

i) Suppose that $z = f(x,y)$ is a differentiable function of $x$ and $y$, where $x = g(t)$ and $y = h(t)$ are both differentiable functions of $t$. Then show that $z$ is a differentiable function of t and

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}.$$

ii) If $f(x,y)$ is a homogeneous function of degree $n$ and $f(x,y)$ has continuous second - order partial derivatives, then show that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = n\,f(x,y).$$

b) Attempt any one of the following. **[5]**

i) Find the shortest distance from the point (1,0,–2) to the plane $x + 2y + z = 4$, using second derivative test for a function of two variables.

ii) Find the extreme values of the function $f(x,y) = x^2+2y^2$ on the circle $x^2+y^2 = 1$ using the method of lagrange multipliers.

*Q4)* a) Attempt any one of the following. **[5]**

i) State fubini's theorem for double integral. Show that if

$f(x,y) = g(x)\, h(y)$ and R = [a,b]×[c,d], then

$$\iint_R f(x,y)\, dA = \int_a^b g(x)dx \int_c^d h(y)dy.$$

ii) State the formula to change rectangular coordinates to polar coordinates in a double integral. Hence evaluate $\iint_R (3x+4y^2)dA$ ,where R is the region in the upper half plane bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$.

b) Attempt any one of the following. **[5]**

i) Sketch the region of integration and change the order of integration $\int_0^1 \int_0^y f(x,y)dx\, dy$.

ii) Evaluate $\int_0^1 \int_x^{2x} \int_0^y 2x\, yz\, dz\, dy\, dx$.

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