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Chapter 1: Solution of Algebraic and Transcendental Equations

Topic- Bisection Method, False Position Method, Newton Raphson Method

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Paper - II (A)

Numerical methods and its applications

Chapter - I

Solution of algebraic & transcendental equations

If $f(x)$ contains $\log x$, $\sin x$, e^x, ... then $f(x)=0$ is called transcendental eqn. e.g. $\sin x$ $x^2 + \tan^{-1} x = 0$
$x^2+1=0 \Rightarrow$

Significant digits or significant figures -

The digits that are used to express a number are called as significant digits or significant figures.

e.g. 1.2345 ⇒ It has 5 sig. fig.

2402347 ⇒ — " — 7 — " —

001 ⇒ it has 1 significant figure

100 ⇒ — " — 3 — " —

Rounding off number - : In numerical computations, we come across the number which have large number of figures in it. for e.g 1.34567892 4. In that case we cut-off the number to suitable size such as 1.2, 3.42, 5.693, etc. Then this process is called as rounding-off number.

Rules for round - off number to n significant figures -

Discard all digits to the right of the n^{th} digit. If $(n+1)^{th}$ digit is

1. less than 5, keep n^{th} digit as it is
2. greater than 5, increase n^{th} digit by 1
3. equal to 5
   1. If n^{th} digit is even number then keep it as it is
   2. — " — odd — " — then increase n^{th} digit by 1

Ex. Round off the following numbers two 2, 3, 4 decimal places

[decimal places means count the digit from decimal point.

	2 D.P	3 D.P	4 D.P
48.2346789	48.23	48.235	48.2347
1.346924	1.35	1.345	1.3469
21.245962	21.24	21.246	21.2460
381.25584	381.26	381.256	381.2558
1.234596	1.23	1.234	1.2346
2.4687523	2.47	2.469 ,	2.4688

2) Round off the following no. two four significant figures

38.46256 $\Rightarrow$ 38.46

0.02456778 $\Rightarrow$ 0.02457

246.3469 $\Rightarrow$ 246.3

19.2469 $\Rightarrow$ 19.25

0.00024638 $\Rightarrow$ 0.0002464

5.00256 $\Rightarrow$ 5.002

5.00357 $\Rightarrow$ 5.004

1) Error - If x is true value & x_1 is an approximate value then error is given by

$$\text{Error} = E = x - x_1$$

2) Absolute error -

The absolute error is denoted by E_A & is given by

$$E_A = |x - x_1| .$$

3] Relative error -

The relative error is denoted by E_R & is given by

$$E_R = \frac{|x - x_1|}{|x|} = \frac{E_A}{|x|} = \frac{\text{absolute error}}{\text{true value}}$$

4) Percentage error -

The percentage error is denoted by E_P & is given by

$$E_P = 100 \times E_R = \frac{|x - x_1|}{|x|} \times 100$$

Remark - Let Δx be a number such that

$$|x - x_1| \leq \Delta x \qquad \text{i.e. absolute error} \leq \Delta x$$

Then Δx is an upper limit on the magnitude of the absolute error and is said to measure absolute accuracy.

similarly, $\dfrac{\Delta x}{|x|} \approx \dfrac{\Delta x}{|x_1|}$

measures the relative accuracy.

Remark - If the number x is rounded to N decimal places then absolute accuracy Δx is

$$\Delta x = \frac{1}{2} \times 10^{-N}$$

& percentage accuracy is $\dfrac{\Delta x}{|x|} \times 100$

Example 1) :- Let $x = 3.1416$ be an actual value &

$x_1 = 3.14159$ be an approximate value

of π then find absolute error, relative error & percentage error.

$\Rightarrow$ 1) absolute error is

$$E_A = |x - x_1| = |3.1416 - 3.14159| = 0.00001$$

2) Relative error is

$$E_R = \frac{E_A}{|x|} = \frac{0.00001}{3.1416} = 0.0000031831$$

3) Percentage error is

$$E_P = E_R \times 100 = 0.0000031831 \times 100$$
$$= 0.00031831\ \%$$

2) If $x = 1.234$ is true value & $x_1 = 1.23456$ is approximate value then find absolute error, relative error & percentage error

$\Rightarrow$ Absolute error

$$E_A = |x - x_1| = |1.234 - 1.23456| = 0.00056$$

Relative error

$$E_R = \frac{E_A}{|x|} = \frac{0.00056}{1.234} = 0.0004538$$

Percentage error

$$E_P = E_R \times 100 = 0.04538\ \%$$

3) If $x = 0.51$ is correct upto two decimal places then find the absolute error (absolute accuracy) Δx and percentage accuracy

$\Rightarrow$ Since $x = 0.51$ is rounded to 2-decimal places

$\therefore$ $N = 2$

$\therefore$ absolute accuracy is

$$\Delta X = \frac{1}{2} \times 10^{-N}$$

$$= \frac{1}{2} \times 10^{-2} = 0.5 \times \frac{1}{100} = \frac{0.5}{100} = 0.005$$

percentage accuracy is $= \frac{\Delta X}{|X|} \times 100$

$$= \frac{0.005}{0.51} \times 100$$

$$= 0.9803 \%$$

4) If $X = 0.25$ & $X_1 = 0.251$ is correct to two decimal places then find the error & absolute accuracy

$\Rightarrow$ $X = 0.25$, $X_1 = 0.251$

$\therefore$ absolute error

$$E_A = |X - X_1| = |0.25 - 0.251| = 0.001$$

Since $X = 0.251$ is correct upto two decimal places

$\therefore N = 2$

$\therefore$ absolute accuracy is

$$\Delta X = \frac{1}{2} \times 10^{-N}$$

$$= \frac{1}{2} \times 10^{-2} = 0.005$$

5) If $X = 0.1416$ & is correct upto four decimal place then find absolute accuracy & percentage accuracy.

$\Rightarrow$ since $X = 0.1416$ is correct upto 4 decimal places

$\therefore N = 4$

$\therefore$ absolute accuracy is

$$\Delta X = \frac{1}{2} \times 10^{-N} = \frac{1}{2} \times 10^{-4} = 0.00005$$

percentage accuracy $= \frac{\Delta X}{|X|} \times 100$

$$= \frac{0.00005}{0.1416} \times 100$$

$$= 0.03531.$$

$$= 0.0353 \%$$

c) Three approximate values of $\frac{1}{3}$ are given as 0.30, 0.33, 0.34 which of these three values is the best approximation?

⇒ Absolute error

$$\left| \frac{1}{3} - 0.30 \right| = \left| \frac{1-0.9}{3} \right| = \left| \frac{0.1}{3} \right| = 0.0333$$

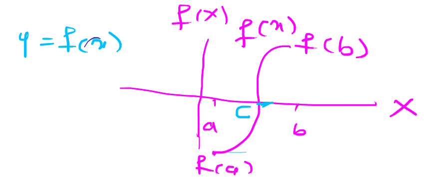
$$\left| \frac{1}{3} - 0.33 \right| = \left| \frac{1-0.99}{3} \right| = \left| \frac{0.01}{3} \right| = 0.0033$$

$$\left| \frac{1}{3} - 0.34 \right| = \left| \frac{1-1.02}{3} \right| = \left| \frac{-0.02}{3} \right| = 0.0067$$

absolute error is less for 0.33. Hence 0.33 is best approximation.

Location of root thm -

If $f(x)$ is cont in $[a, b]$ & if $f(a)$ & $f(b)$ has opposite signs then $f(x) = 0$ has atleast one real root betn a & b



$f(c) = 0$
$\Rightarrow c$ is root

Bisection Method -

$-ve = x_0$
$+ve = x_1$

1) choose x_0 & x_1 s.t. $f(x_0) < 0$ & $f(x_1) > 0$
2) Take $x_2 = \dfrac{x_0 + x_1}{2}$
3) If $f(x_2) = 0$ then x_2 is root of given eqn
   If $f(x_2) < 0$ then take $x_0 = x_2$
   If $f(x_2) > 0$ then take $x_1 = x_2$

   repeate the steps 2 & 3 untill we get root of require accuracy.

4) The convenient criteria is to compute the porcentage error ϵ_r is

$$\epsilon_r = \left| \frac{x_r' - x_r}{x_r'} \right| \times 100 \%$$

where x_r' is the new value of x_r. The computation can be terminated when ϵ_r become less than given tolerance.

Example 1) Find the real root of the equation $f(x) = x^3 - x - 1 = 0$ correct upto three decimal places.

$\Rightarrow$ $f(0) = -1 < 0$, $f(1) = -1 < 0$, $f(1.5) = 0.875 > 0$

$\therefore$ root lies betn 1 & 1.5

Take $x_0 = 1$ & $x_1 = 1.5$

$-ve = x_0$
$+ve = x_1$

1) $x_2 = \dfrac{x_0 + x_1}{2} = \dfrac{1 + 1.5}{2} = \dfrac{2.5}{2} = 1.25$

$f(x_2) = (1.25)^3 - 1.25 - 1 = -0.2969 < 0$

$\therefore f(x_2) < 0$ $\therefore$ Take $x_0 = x_2 = 1.25$, $x_1 = 1.5$

2) $x_2 = \dfrac{x_0 + x_1}{2}$

$= \dfrac{1.25 + 1.5}{2} = 1.375$

$f(x_2) = (1.375)^3 - (1.375) - 1 = 0.2246 > 0$

since $f(x_2) > 0$ $\therefore$ Take $x_1 = x_2 = 1.375$, $x_0 = 1.25$

3) $x_2 = \dfrac{x_0 + x_1}{2}$

$= \dfrac{1.25 + 1.375}{2} = 1.3125$

$f(x_2) = -0.0515 < 0$

$\therefore$ Take $x_0 = 1.3125$, $x_1 = 1.375$

4) $x_2 = \dfrac{x_0 + x_1}{2}$

$= \dfrac{1.3125 + 1.375}{2} = 1.3438$

$f(x_2) = 0.0828 > 0$

Take $x_1 = 1.3438$, $x_0 = 1.3125$

5) $x_2 = \dfrac{x_0 + x_1}{2} = \dfrac{1.3438 + 1.3125}{2} = 1.3282$

$f(x_2) = 0.0149 > 0$

Take $x_1 = x_2 = 1.3282$, $x_0 = 1.3125$

6) $x_2 = \dfrac{x_0 + x_1}{2} = \dfrac{1.3125 + 1.3282}{2} = 1.3204$

$f(x_2) = -0.0183 < 0$

Take $x_0 = x_2 = 1.3204$, $x_1 = 1.3282$

7) $x_2 = \dfrac{x_0 + x_1}{2} = \dfrac{1.3204 + 1.3282}{2} = 1.3243$

$f(x_2) = -0.0018 < 0$

$\therefore$ Take $x_0 = x_2 = 1.3243$, $x_1 = 1.3282$

8) $x_2 = \dfrac{x_0 + x_1}{2} = \dfrac{1.3243 + 1.3282}{2} = 1.3262$

$f(x_2) = 0.0063 > 0$

$\therefore$ Take $x_1 = x_2 = 1.3262$, $x_0 = 1.3243$

9) $x_2 = \dfrac{x_0 + x_1}{2} = \dfrac{1.3262 + 1.3243}{2} = 1.3252$

$f(x_2) = 0.00206 > 0$

$\therefore$ Take $x_1 = x_2 = 1.3252$ & $x_0 = 1.3243$

10) $x_2 = \frac{x_0 + x_1}{2} = \frac{1.3243 + 1.3252}{2} = 1.3248$

$f(x_2) = 0.000035 > 0$

$\therefore$ Take $x_1 = x_2 = 1.3248$, $x_0 = 1.3243$

11) $x_2 = \frac{x_0 + x_1}{2} = \frac{1.3243 + 1.3248}{2} = 1.3246$

$f(x_2) = -0.0005 \approx 0$

since 1.3248 & 1.3246 are correct upto 3 decimal places

$\therefore$ $x_2 = 1.3246$ is approximate root of given eqn.

Method of False Position – (Linear interpolation method or Regula Falsi method)

If we have to find root of $f(x)=0$ then choose any two point say x_0 & x_1 s.t. $f(x_0)<0$ & $f(x_1)>0$ then

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{x_0 + x_1}{2}$$

or

$$x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)}$$

case i) If $f(x_2)=0$ then x_2 is root of $f(x)=0$

case ii) If $f(x_2)<0$ then take $x_0=x_2$

case iii) If $f(x_2)>0$ then take $x_1=x_2$

& repeate the process until we get root of required accuracy.

Ex. Find the real root correct to three decimal places by the method of False-Position

$$f(x) = x^3 - x - 4 = 0$$

$\Rightarrow$ $f(1) = 1-1-4 = -4 < 0$, $f(2) = 8-2-4 = 2$

$f(1.5) = -2.125 < 0$, $f(1.8) = 0.032 > 0$

Take $x_0 = 1.5$, $x_1 = 1.8$

$f(x_0) = -2.125 < 0$ & $f(x_1) = 0.032 > 0$

$\therefore$ root lies bet 1.5 & 1.8

i) $$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{1.5(0.032) - 1.8(-2.125)}{0.032 - (-2.125)}$$

$$= \frac{0.048 + 3.825}{2.157} = 1.7955$$

$f(x_2) = -0.0071 < 0$

$f(x_2) < 0$ Take $x_0 = x_2 = 1.7955$, $x_1 = 1.8$

$f(x_0) = -0.0071$ & $f(x_1) = 0.032$

2) $x_i = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{1.7955 (0.032) - 1.8 (-0.0071)}{0.032 - (-0.0071)}$

$= \dfrac{0.0574 + 0.0128}{0.0391} = \dfrac{0.0702}{0.0391} = 1.7954$

$f(x_2) = f(1.7954) = -0.0079 \approx 0$

$\therefore$ $x_2 = 1.7954$ is root of $f(x) = 0$ correct upto three decimal places.

3] $x^3 - 3x + 1 = 0$ find root of $f(x) = 0$, by using False position method.

$\Rightarrow$ $f(x) = x^3 - 3x + 1$

, $f(1) = 1 - 3 + 1 = -1$, $f(2) = 8 - 6 + 1 = 3$

$f(1.5) = -0.125$, $f(1.6) = 0.296$

Take $x_0 = 1.5$ & $x_1 = 1.6$

$f(x_0) = -0.125$ & $f(x_1) = 0.296$

1) $x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{1.5 (0.296) - 1.6 (-0.125)}{0.296 - (-0.125)}$

$= \dfrac{0.4444 + 0.2}{0.421}$

$= \dfrac{0.6444}{0.421} = 1.5297$

$f(x_2) = -0.0096 < 0$

$f(x_2) < 0$ $\therefore$ take $x_0 = x_2 = 1.5297$, $x_1 = 1.6$

$f(x_0) = -0.0096$, $f(x_1) = 0.296$

2) $x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{1.5297 (0.296) - 1.6 (-0.0096)}{0.296 - (-0.0096)}$

$$= \frac{0.4528 + 0.0154}{0.3056}$$

$x_2 = 1.5321$

$f(x_2) = 0.000045 \approx 0$

$\therefore x_2 = 1.5321$ is approximate root of given eqn.

3] Find a real root of the eqn $f(x) = x\tan x + 1 = 0$ in $(2,3)$ correct upto three decimal places

$\Rightarrow f(2.7) = -0.2764 < 0$ $f(2.8) = 0.0045 > 0$

$\therefore$ root lies betn 2.7 & 2.8

Take $x_0 = 2.7$ & $x_1 = 2.8$

$f(x_0) = -0.2764$, $f(x_1) = 0.0045$

tan 2.7

$\Rightarrow$ tan 2.7 + shift + Ans + 2 =

(i) $$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{2.7(0.0045) - 2.8(-0.2764)}{0.0045 + 0.2764}$$

$$= \frac{0.0122 + 0.7739}{0.2809}$$

$x_2 = 2.7985$

$f(x_2) = 0.0003 > 0$

$\therefore$ Take $x_1 = 2.7985$, $x_0 = 2.7$

$f(x_1) = 0.0003$, $f(x_0) = -0.2764$

(ii) $$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{2.7(0.0003) - 2.7985(-0.2764)}{0.0003 + 0.2764}$$

$$= \frac{0.0008 + 0.7735}{0.2767}$$

$x_2 = 2.7983$

$$f(x_2) = -0.0002 \approx 0$$

$\therefore$ $x_2 = 2.7983$ is approximate root correct upto three decimal places.

Thm - Prove that the Newton - Raphson method has second order or quadratic convergence.

Proof - Let ξ be the root of $f(x) = 0$

$$\therefore f(\xi) = 0$$

$$f(\underset{x}{x_n} + \underset{h}{(\xi - x_n)}) = 0$$

Taylor's series is

$$f(x_n) + (\xi - x_n) f'(x_n) + \frac{(\xi - x_n)^2}{2!} f''(x_n) + \dots = 0$$

$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \dots$

neglecting 3rd & higher order derivatives

$$\therefore f(x_n) + (\xi - x_n) f'(x_n) + \frac{(\xi - x_n)^2}{2} f''(x_n) = 0$$

$$f(x_n) = -(\xi - x_n) f'(x_n) - \frac{(\xi - x_n)^2}{2} f''(x_n)$$

$$\frac{f(x_n)}{f'(x_n)} = -(\xi - x_n) - \frac{1}{2}(\xi - x_n)^2 \frac{f''(x_n)}{f'(x_n)}$$

By Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow \frac{f(x_n)}{f'(x_n)} = x_n - x_{n+1}$$

$$\cancel{x_n} - x_{n+1} = -\xi + \cancel{x_n} - \frac{1}{2}(\xi - x_n)^2 \frac{f''(x_n)}{f'(x_n)}$$

$$\xi - x_{n+1} = -\frac{1}{2}(\xi - x_n)^2 \frac{f''(x_n)}{f'(x_n)}$$

multiply by -1

$$x_{n+1} - \xi = \frac{1}{2}(\xi - x_n)^2 \frac{f''(x_n)}{f'(x_n)}$$

put $\epsilon_n = x_n - \xi$, $\epsilon_{n+1} = x_{n+1} - \xi$

$$\epsilon_{n+1} = \frac{1}{2} \epsilon_n^2 \frac{f''(x_n)}{f'(x_n)}$$

$\therefore$ This shows that Newton Raphson method has quadratic convergence.

Ex. obtain Newton-Raphson formula to find (i) square root (ii) r^{th} root of given number.

$\Rightarrow$ (i) suppose we have to find square root of c

i.e $\sqrt{c}$

Take $x = \sqrt{c}$

$\Rightarrow x^2 = c$

$\Rightarrow x^2 - c = 0$

Take, $f(x) = x^2 - c$.

$f'(x) = 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_n^2 - c)}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 + c}{2x_n}$$

$$x_{n+1} = \frac{x_n^2 + c}{2x_n}$$

sq root $c^{\frac{1}{2}}$

cube root $c^{\frac{1}{3}}$

4^{th} root $c^{\frac{1}{4}}$

r^{th} root $c^{\frac{1}{r}}$

(ii) r^{th} root

We have to find out $c^{\frac{1}{r}}$

Let $x = c^{\frac{1}{r}}$

$\Rightarrow x^r = c$

$\Rightarrow x^r - c = 0$

consider, $f(x) = x^r - c$

$f'(x) = r x^{r-1}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_n^r - c)}{r x_n^{r-1}}$$

$$= \frac{r x_n^r - x_n^r + c}{r x_n^{r-1}}$$

$$x_{n+1} = \frac{(r-1)x_n^r + c}{r x_n^{r-1}}$$

Example – Find a real root which lies betn 2 & 3 of the eqn $x\log_{10}x - 1.2 = 0$ using the method of False position to a tolerance 0.5%. (or 0.005)

$\Rightarrow f(2.5) = 2.5\log 2.5 - 1.2 = -0.2051 < 0$ $\quad \log_{10} x = \log$

$f(2.7) = -0.0353 < 0$

$f(2.8) = 0.0520 > 0$ $\quad \log_e x = \ln$

Take $x_0 = 2.7$ & $x_1 = 2.8$

$f(x_0) = -0.0353 < 0$ & $f(x_1) = 0.0520 > 0$

① $$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{2.7(0.0520) - 2.8(-0.0353)}{0.0520 + 0.0353}$$

$$= \frac{0.1405 + 0.0988}{0.0873}$$

$x_2 = 2.7416$

$f(x_2) = 0.0008 > 0$

Take $x_1 = 2.7416$ & $x_0 = 2.7$

$f(x_1) = 0.0008$, $f(x_0) = -0.0353$

2) $$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{2.7(0.0008) - 2.7416(-0.0353)}{0.0008 + 0.0353}$$

$$= \frac{0.0022 + 0.0968}{0.0361}$$

$x_2 = 2.7424$

$f(x_2) = 0.0015 \approx 0$

Percentage error $\epsilon_p = \left| \frac{x_2' - x_2}{x_2'} \right| \times 100$, where x_2' is new value of x_2

$$\epsilon_p = \left| \frac{2.7424 - 2.7416}{2.7424} \right| \times 100$$

$= 0.0291\% < 0.05\%$.

$\therefore x_2 = 2.7424$ is approximate root to tolerance 0.5%.

Ex. Find a real the smallest positive root of the eqn $x^4 - x - 10 = 0$ correct to three decimal places using the method of False position to a tolerance of 0.005 (0.005 means 0.5%)

$\Rightarrow f(x) = x^4 - x - 10$

$f(1) = 1 - 1 - 10 = -10$, $f(2) = 16 - 2 - 10 = 4$

$f(1.8) = -1.3024 < 0$, $f(1.9) = 1.1321 > 0$

$\therefore$ Take $x_0 = 1.8$, $x_1 = 1.9$

$f(x_0) = -1.3024$, $f(x_1) = 1.1321$

1) $x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \frac{1.8(1.1321) - 1.9(-1.3024)}{1.1321 + 1.3024}$

$= \frac{2.0378 + 2.4746}{2.4345}$

$x_2 = 1.8535$

$f(x_2) = -0.0511 < 0$,

$\therefore$ Take $x_0 = 1.8535$, $x_1 = 1.9$

$f(x_0) = -0.0511$, $f(x_1) = 1.1321$

2) $x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$$= \frac{1.8535(1.1321) - 1.9(-0.0511)}{1.1321 + 0.0511}$$

$$= \frac{2.0983 + 0.0971}{1.1832}$$

$x_2 = 1.8555$

$f(x_2) = -0.0021 < 0$

$\therefore$ Take $x_0 = 1.8555$, $x_1 = 1.9$

$f(x_0) = -0.0021$, $f(x_1) = 1.1321$

3) $$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{2.1006 + 0.00399}{1.1342}$$

$x_2 = 1.8556$

$f(x_2) = 0.0004 \approx 0$

root is correct upto three decimal places

also

$$\epsilon_p = \left| \frac{x_2' - x_2}{x_2'} \right| \times 100,$$ x_2' is new value of x_2

$$= \left| \frac{1.8556 - 1.8555}{1.8556} \right| \times 100$$

$$= 0.005\% < 0.5\%$$

$\therefore$ $x_2 = 1.8556$ is approximate root to a tolerance 0.005 i.e 0.5%.

Reference: Numerical methods and its applications, text book for S.Y.B.Sc., by Golden series, Nirali Prakashan.