K.T.S.P. Mandal's

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Subject- Classical Electrodynamics

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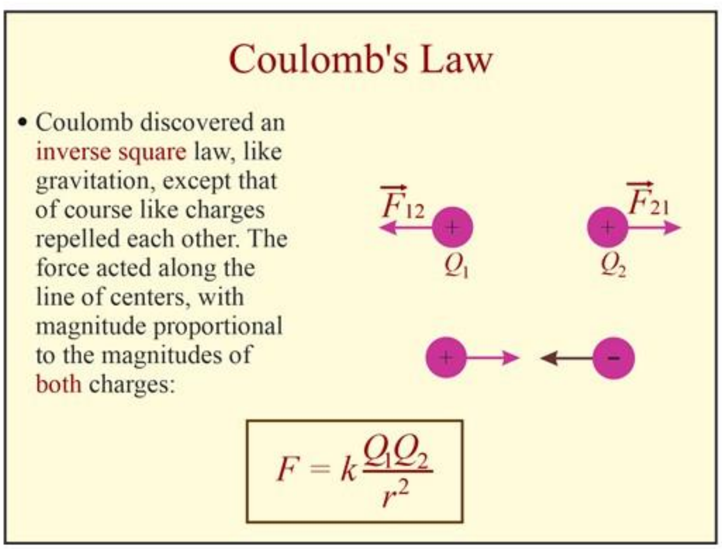
Topic- Electrostatics

❖ Coulomb's Law:

Coulomb's law defines the relationship between the electrostatic force of attraction and repulsion between two electrically charged bodies. According to this Law, the magnitude of the electrostatic force between two charged bodies is:

1. Directly proportional to the product of the magnitude of charges on the two bodies.
2. Inversely proportional to the square of the distance between the centres of the charged bodies.

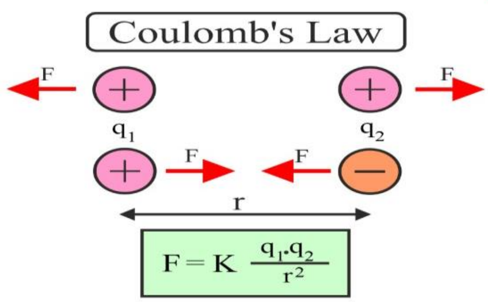
Coulomb's law acts along the line joining the centres of the two charged bodies. Since the relation describes an inverse square relation between force and distance between the two charged bodies, that is why the law is also referred to as Coulomb's inverse-square law. The relation derived from Coulomb's Law is quite similar to the gravitational force acting between two massive bodies; that is why Coulomb's Law is considered as an electrical analogue of Newton's Universal Law of Gravitation.



This law gave a perfect estimate of the force acting between two point charges. In physics, the word "Point charge" is used to define extremely small bodies. This means that the linear size of the charged bodies is very small, especially in comparison to the distance between the two bodies. Thus, by considering these charged bodies as point charges, it becomes easier for us to calculate the magnitude of electrostatic forces acting between them.

Understanding Coulomb's Law

From Coulomb's Law, we can conclude that charges of the same sign will push each other way due to the forces of repulsion acting between them. In contrast, charges with opposite polarities will pull each other closer due to the force of attraction acting between them. This can be stated as like charges repel each other and unlike charges attract each other.



To visualise this, imagine that there are two equal and opposite charges separated by a small distance. The two charges will attract each other. Now, if the two charges are brought closer, the force of attraction between the two will get stronger.

If the charges are moved apart, the force of attraction between the two will decrease. Thus, we can say that the force between two charged bodies varies inversely with the distance between the two charges. If dd is the distance between the charged bodies, then according to Coulomb's Law, the electrostatic force between them varies as:

$$F \propto 1/d^2$$

Keeping these charges fixed, if the magnitude of the charge is now increased, the force of attraction between the charges increases and on decreasing the magnitude of these charges, we find that the force of attraction between the charges decreases. From Coulomb's Law, if q1q1 and q2q2 be the magnitude of the charges, then:

$$F \propto q_1 q_2$$

This force is affected by the medium in which the charges are kept. If ε represents the property of a medium, then according to Coulomb's Law,

$F \propto 1/\varepsilon$

Derivation of Coulomb's Formula

Coulomb's Law statement helps us understand the relationship between charge and distance and how it influences the electrostatic force (i.e. the electric force between charged bodies at rest). This force is also known as Coulomb's force. To calculate the expression of Coulomb's force, Let that there be two charges *Q*1 and *Q*2, if these two charges are kept at a distance *r* from each other, the force of attraction/repulsion between them is *F*. Then:

$F \propto Q1Q2$ ....(i)

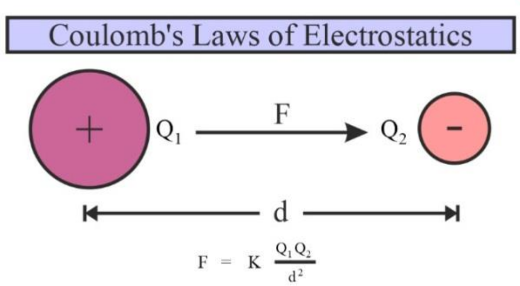
$F \propto 1/r^2$ ....(ii)

Combine equations (i) and (ii), we get

$F \propto (Q1Q2)/r^2$

$F = k\, Q1Q2 / r^2$

Here, k is the constant of proportionality.



$k = 1 / 4\pi\varepsilon_0$

Thus,

$F = 1/ 4\pi\varepsilon0 . Q1Q2 / r^2$

Where, ε_0 represents the absolute permittivity of the free space or vacuum.

In terms of the S.I unit, the value of $\varepsilon_0 = 8.85\times10^{-12}\ C^2 / N\ m^2$

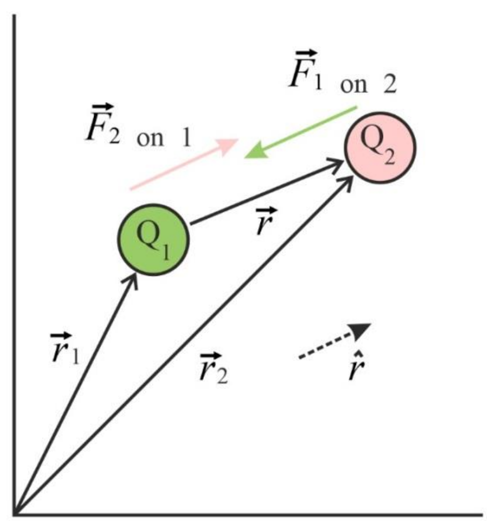
Thus,

Vector Form of Coulomb's Law

We know that all physical quantities can be categorised into two Scalars or Vectors. Scalars are quantities with the only magnitude, while vectors are quantities with both direction and magnitude. Force is a vector quantity. Thus, it will be represented by an arrow over it.

To write the expression for Coulomb's Law in its vector form, let there be two charges Q_1 and Q_2, such that $\vec{r}_1$ and $\vec{r}_2$ represent the position vectors of the two charges, respectively.

The two charges will exert electrostatic forces on each other. Let $\vec{F}_{12}$ be the force exerted by the charge Q_1 on Q_2 and $\vec{F}_{21}$ be the force exerted by the charge Q_2 on Q_1. Suppose the corresponding vector from Q_1 to Q_2 is given by $\vec{r}_{21}$.



Thus, using triangle law,

$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$$

The direction of position vector from $\vec{r}_1$ to $\vec{r}_2$ and $\vec{r}_2$ to $\vec{r}_1$, can be given as:

$$\vec{r}_{21} = \vec{r}_{21} / |\vec{r}_{21}|$$

$\hat{r}_{12} = \vec{r}_{12} / |\vec{r}_{12}|$

Therefore, the force acting on the charge Q_1 due to Q_2, in vector form can be given as:

$\vec{F}_{21} = 1 / 4\pi\varepsilon 0. * Q_1Q_2 / |\vec{r}_{21}|^2 * \hat{r}_{21}$

The above equation is the vector form of Coulomb's Law.

Limitations of Coulomb's Law

1. The validity of Coulomb's Law is determined by the number of molecules of solvent between the two charged bodies. If the average number of solvent molecules between the two interacting charged particles is large enough, we can only apply this law.
2. We can only apply Coulomb's Law on stationary charges.
3. If the shape of the charged body is arbitrary, then we will not be able to determine the distance between the charges, and it becomes difficult to apply this law.

❖ Electric Field:

The electric field $\vec{E}$ is a vector quantity that exists at every point in space. The electric field at a location indicates the force that *would* act on a unit positive test charge *if* placed at that location.

The electric field is related to the electric force that acts on an arbitrary charge q by,

$\vec{E} = \vec{F} / q$

The dimensions of electric field are newtons/coulomb, N/C.

We can express the electric force in terms of electric field,

$\vec{F} = \vec{q} * \vec{E}$

For a positive q, the electric field vector points in the same direction as the force vector.

a) Electric intensity due to point charge :

The electric field direction points straight away from a positive point charge, and straight at a negative point charge. The magnitude of the electric field falls off as $1/r^2$ going away from the point charge.

The electric field around a single isolated point charge, q_given by,

$$E = 1/ 4\pi\epsilon_0 * q / r^2 * \hat{r}$$

Electric field near multiple point charges

If we have multiple charges scattered about, we express the electric field by summing the fields from each individual q_i,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r^2} \hat{r}_i$$

The summation is performed as a vector sum.

Electric field near distributed charge

If charges are smeared out in a continuous distribution, the summation evolves into an integral.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

Where r is the distance between dq and the location of interest, while $\hat{r}$ reminds us the direction of the force is in a direct line between dq and the location of interest. We will see examples of this integral in two upcoming articles.

The discussion above defines the electric field. There isn't any new physics, we've just defined some new terms. Now we're ready to move on and use the electric field formulation to analyze two common real-world geometries: the line of charge, and the plane of charge.

The charge density is the measure for the accumulation of electric charge in a given particular field. It measures the amount of electric charge as per the following dimensions:

(i) Per unit length i.e. linear charge density, where q is the charge and is the length over which it is distributed. The SI unit will be Coulomb m^{-1}.

Linear charge density is computed as:

$$\lambda = \Delta q / \Delta l$$

(ii) Per unit surface area i.e. surface charge density, where, q is the charge and A is the area of the surface. The SI unit is Coulomb m^{-2}.

$$\sigma = \Delta q / \Delta A$$

(iii) Per unit volume i.e. volume charge density, where q is the charge and V is the volume of distribution. The SI unit is Coulomb m^{-3}.

$$\rho = \Delta q / \Delta V$$

❖ GAUSS'S LAWS

Without Gauss Theorem study of electrostatic is incomplete. So it became necessary to know about **gauss theorem**. In this article we are going to discuss about gauss theorem in detail. So keep reading till end.

In **electromagnetism**, **gauss's law** is also known as **gauss flux's theorem.** It is a law which relates the distribution of electric charge to the resulting electric field. Gauss's Law is mathematically very similar to the other laws of physics. Such as – gauss's law for magnetism, gauss's law for gravity.

However, gauss's law can be expressed in such a way that it is very similar to the inverse square law. In physics there are many inverse square laws, such as inverse square coulomb's law, inverse square Newton's laws of gravity , inverse square magnetic coulomb's law etc. Gauss's law is essentially very equivalent or analogies to these inverse square laws.

There are two forms in which gauss's law can be expressed. We can express this law mathematically using vector calculus in integral form and in differential form, both form are equivalent since they are related by the divergence theorem which are also called Gauss theorem.

1) Integral form – This form of gauss theorem is used when distribution of electric charge on any closed surface is continuous or symmetrical. Integral form of gauss law states that electric flux through any arbitrary surface is proportional to the total electric charge enclosed by the surface.

2). **Differential form** – This form of gauss theorem is used when distribution of electric charge over a closed surface is discontinious or unsymmetrical. Differential form of gauss law states that the divergence of the electric field is proportional to the local density of charge.

INTEGRAL FORM

GAUSS'S LAW : STATEMENT

The gauss theorem states that – The surface integral of the electric field intensity over a closed surface (called Gaussian surface) in free space is equal to the times the net electric charge enclosed within the surface. Mathematically it is given as –

$$\Phi_E = \oint_S E.dS = \frac{1}{\epsilon_0}\sum_{i=1}^{n} qi = \frac{q}{\epsilon_0}$$

Where $\sum_{i=1}^{n} qi = \frac{q}{\epsilon_0}$ is the algebraic sum of all the charge inside the closed surface. Hence, the total electric flux over a closed surface in vaccum is times the total charge within the surface irrespective of how that charge is distributed.

Differential Form of Gauss' Law:

Recall that Gauss' Law says that

$$\int_{\text{box}} \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} Q_{\text{inside}}.$$

But the enclosed charge is just

$$Q_{\text{inside}} = \int_{\text{box}} \rho \, dV$$

so we have

$$\int_{\text{box}} \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \int_{\text{box}} \rho \, dV.$$

Applying the Divergence theorem to the left-hand side of this equation tells us that

$$\int_{\text{inside}} \vec{\nabla} \cdot \vec{E} \, dV = \frac{1}{\epsilon_0} \int_{\text{box}} \rho \, dV$$

for *any* closed box. This means that the integrands themselves must be equal, that is,

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}.$$

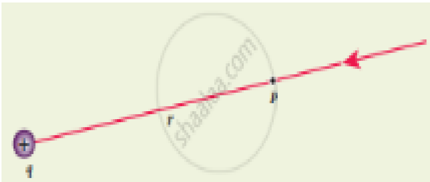
This conclusion is the differential form of Gauss' Law, and is one of Maxwell's Equations. It states that the divergence of the electric field at any point is just a measure of the charge density there.

❖ Electric Potential

Electric potential is defined as the amount of work needed to move a unit charge from a reference point to a specific point against the electric field. When an object is moved against the electric field it gains some amount of energy which is defined as the electric potential energy. The electric potential of the charge is obtained by dividing the potential energy by the quantity of charge.

Strength of the electric field depends on the electric potential. It is independent of the fact whether a charge should be placed in the electric field or not. Electric potential is a scalar quantity. At point charge +q there is always the same potential at all points with a distance r. Let us learn to derive an expression for the electric field at a point due to a system of n point charges.

1. Consider a positive charge ‘q’ kept fixed at the origin. Let P be a point at distance r from the charge ‘q’.



Electrostatic potential at a point P

2. The electric potential at the point P is

$$V = \int_{\infty}^{r} \left(-\vec{E}\right) \cdot \vec{dr} = -\int_{\infty}^{r} \vec{E} \cdot \vec{dr}$$

3. Electric field due to positive point charge is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

$$V = -\frac{1}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{q}{r^2} \hat{r} \cdot \vec{dr}$$

4. The infinitesimal displacement vector,

$\vec{dr} = dr\hat{r}$ and using $\hat{r}\hat{r} = 1$, we have

$$V = -\frac{1}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{q}{r^2} \hat{r} \cdot dr\hat{r} = -\frac{1}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{q}{r^2} dr$$

5. After the integration,

$$V = -\frac{1}{4\pi\varepsilon_0} q \left\{-\frac{1}{r}\right\}_{\infty}^{r} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

6. Hence the electric potential due to a point charge q at a distance r is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

❖ Electric Potential Energy :

The electric potential energy of any given charge or system of changes is termed as the total work done by an external agent in bringing the charge or the system of charges from infinity to the present configuration without undergoing any acceleration.

Definition: Electric potential energy is defined as the total potential energy a unit charge will possess if located at any point in the outer space.

Electric potential energy is a scalar quantity and possesses only magnitude and no direction. It is measured in terms of Joules and is denoted by V. It has the dimensional formula of $ML^2T^{-3}A^{-1}$.

A charge placed in an electric field possesses potential energy and is measured by the work done in moving the charge from infinity to that point against the electric field. If two charges q_1 and q_2 are separated by a distance d, the e**lectric potential energy of the system** is;

$$U = [1/(4\pi\varepsilon_o)] \times [q_1q_2/d]$$

If two like charges (two protons or two electrons) are brought towards each other, the potential energy of the system increases. If two unlike charges i.e. a proton and an electron are brought towards each other, the electric potential energy of the system decreases.

Electric Potential Energy of a System:

The electrostatic potential between any two arbitrary charges q_1, q_2 separated by distance r is given by Coulomb’s law and mathematically written as:
$U = k \times [q_1q_2/r^2]$

Where,

- U is the electrostatic potential energy,
- q_1 and q_2 are the two charges.

Note: The electric potential is at infinity is zero (as, r = ∞ in the above formula).

Let us consider a charge q_1. Let us say, they are placed at a distance ‘r’ from each other. The total electric potential of the charge is defined as the total work done by an external force in bringing the charge from infinity to the given point.

We can write it as, $-\int (r_a \rightarrow r_b)$ F.dr = – $(U_a - U_b)$

Here, we see that the point r_b is present at infinity and the point r_a is r.

Substituting the values we can write, $-\int (r \rightarrow \infty)$ F.dr = – $(U_r - U_\infty)$

As we know that U_{infity} is equal to zero.

Therefore, $-\int (r \rightarrow \infty)$ F.dr = $-U_R$

Using Coulomb's law, between the two charges we can write:

$\Rightarrow -\int (r \rightarrow \infty) [-kqq_o]/r^2 \, dr = -U_R$

Or, $-k \times qq_o \times [1/r] = U_R$

Therefore, $\mathbf{U_R = -kqq_o/r}$

Electric Potential of a Point Charge

Let us consider a point charge 'q' in the presence of another charge 'Q' with infinite separation between them.

$U_E (r) = k_e \times [qQ/r]$

where, $k_e = 1/4\pi\varepsilon_o$ = Columb's constant

Let us consider a point charge 'q' in the presence of several point charges Q_i with infinite separation between them.

$$\mathbf{U_E (r) = k_e\, q \times \sum^{n}_{i = 1} [Q_i / r_i]}$$

Electric Potential for Multiple Charges

In the case of 3 Charges:

If three charges q_1, q_2 and q_3 are situated at the vertices of a triangle, the potential energy of the system is,

$\mathbf{U = U_{12} + U_{23} + U_{31}} = (1/4\pi\varepsilon_o) \times [q_1q_2/d_1 + q_2q_3/d_2 + q_3q_1/d_3]$

In the case of 4 Charges:

If four charges q_1, q_2, q_3 and q_4 are situated at the corners of a square, the electric potential energy of the system is,

$U = (1/4\pi\varepsilon_o) \times [(q_1q_2/d) + (q_2q_3/d) + (q_3q_4/d) + (q_4q_1/d) + (q_4q_2/\sqrt{2}d) + (q_3q_1/\sqrt{2}d)]$

Special Case:

In the field of a charge Q, if a charge q is moved against the electric field from a distance 'a' to a distance 'b' from Q, the work done is given by,

$W = (V_b - V_a) \times q = [1/4\pi\varepsilon_o \times (Qq/b)] - [1/4\pi\varepsilon_o \times (Qq/a)] = Qq/4\pi\varepsilon_o[1/b - 1/a] = (Qq/4\pi\varepsilon_o)[(a-b)/ab]$