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F.Y.B.Sc.
MT-111: Algebra
(2019 Pattern) (Semester-I) (Paper-I) (11111)

Time: 2 Hours **Max. Marks: 35**

Instructions to the candidates:

1) **All questions are compulsory.**
2) **Figures to the right indicate full marks.**

Q1) Attempt any five of the following: **[5]**

a) Express the empty set as a subset of $\mathbb{R}$.

b) Define equivalence relation.

c) If $a|b$ then show that $a|bc$ for any integer c.

d) Find the value of $\overline{15}$ in $\mathbb{Z}_5$.

e) Show that $a \equiv a \ (mod\ n)$.

f) Verify that $z = 1 + i$ satisfy the equation $z^2 - 2z + 2 = 0$.

g) Evaluate $\frac{1+2i}{3-4i}$.

Q2) A) Attempt any one of the following **[5]**

a) Let $\sim$ be an equivalence relation on a nonempty set X. If $y \in [x]$ then show that $[x] = [y]$.

b) Given integers $a\ and\ b$ with $b \neq 0$ there exist unique integers $q\ and\ r$ satisfying $a = bq + r$, where $0 \leq r < |b|$.

B) Attempt any one of the following **[5]**

a) Prepare the composition table for addition and multiplication in $\mathbb{Z}_7$.

b) Let $\sim$ be the relation defined on $\mathbb{R}$ by $x{\sim}y$ if and only f $|x| = |y|$.

Q3) A) Attempt any one of the following **[5]**

a) Let $a\ and\ b$ be integers, not both zero. Then $a\ and\ b$ are relatively prime if and only if there exist integers $x\ and\ y$ such that $1 = ax + by$.

b) Let X be a nonempty set and $\sim$ be an equivalence relation on X. Let $x, y \in X$. Then exactly one of the following is true

i. $[x] = [y] = \emptyset$

ii. $[x] = [y]$.

B) Attempt any one of the following **[5]**

a) Find gcd(12378, 3054) and express it in the form $12378\,x + 3054\,y$ for some integers.

b) Prove that following using Mathematical induction

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}; for\ all\ n \geq 1.$$

Q4) **A) Attempt any one of the following** **[5]**

a) If $a \equiv b(mod\ n)$ and $c \equiv d\ (mod\ n)$ then prove that
$a + c \equiv b + d(mod\ n)$ and $ac \equiv bd(mod\ n)$

b) Let θ be any real number and n be an integer. Then
$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta.$$

B) Attempt any one of the following **[5]**

a) Find the unit digit of 3^{100} by the use of Fermat's theorem.

b) Find the square roots of the $1 - \sqrt{3}i$.