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F.Y.B.Sc.
MT-111: Algebra
(2019 Pattern) (Semester-I) (Paper-I) (11111)

Time: 2 Hours **Max. Marks: 35**

Instructions to the candidates:

1) **All questions are compulsory.**
2) **Figures to the right indicate full marks.**

Q1) Attempt any five of the following: **[5]**

a) Let $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=2x+3$.Show that f is one-one,
b) Define equivalence relation.
c) Let $f,g:\mathbb{R}\to\mathbb{R}$ be defined as $f(x)=x^2$ and $g(x)=2x+3$. Find $(g\circ f)(x)$.
d) Find the value of $\overline{17}$ in $\mathbb{Z}_3$.
e) State Fermat's theorem.
f) Solve the equation $z^2+z+1=0$.
g) If $z=2+3i$ find $\bar{z}\ and\ |z|$.

Q2) A) Attempt any one of the following **[5]**

a) Let X be a nonempty set and $\sim$ be an equivalence relation on X. Let $x,y\in X$. Then exactly one of the following is true
   i. $[x]=[y]=\emptyset$
   ii. $[x]=[y]$.
b) If $a\ and\ b$ are integers, not both zero then there exists a unique positive gcd of $a\ and\ b$ which can be expressed in the form $\gcd(a,b)=ax_0+by_0$.

B) Attempt any one of the following **[5]**

a) Let $A=\{1,2,3\}$ determine which of the relation of A are reflexive, symmetric, transitive

$$R_1=\{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3)\}$$
$$R_2=\{(1,1),(2,2),(3,3)\}$$

b) Find gcd(1819, 3587) and express it in the form $1819\,m+3587\,n$ for some integers.

Q3) A) Attempt any one of the following **[5]**

a) Let $\sim$ be an equivalence relation on a nonempty set X. If $y\in[x]$ then show that $[x]=[y]$.
b) If $a|c$ and $b|c$ with $(a,b)=1$ then show that $ab|c$.

B) Attempt any one of the following **[5]**

a) If n is an odd number then n^2-1 is divisible by 8.
b) Find the remainder when 2^{50} is divided by 7.

Q4) A) Attempt any one of the following **[5]**

a) Let θ be any real number and n be an integer. Then

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta.$$

b) Let P be a prime and suppose that $p \nmid a$. Then $a^{p-1} \equiv 1\ (mod\ p)$.

B) Attempt any one of the following **[5]**

a) Prepare the composition table for addition and multiplication in $\mathbb{Z}_6$.

b) Find expression for $\cos^6\theta\ and \sin^6\theta$ in terms of cosine and sine of multiples of θ.