K.T.S.P. Mandal's

Hutatma Rajguru Mahavidyalaya Rajgurnagar-410505

First Year B. Science

I– Sem (2019 CBCS Pattern)
As per the new syllabus

Subject- Mechanics and Properties of Matter

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Topic- Fluid Mechanics

❖ Concept of Viscous Force , Viscosity and Coefficient of Viscosity :

Viscosity is defined as the degree up to which a fluid resists the flow under an applied force; it is measured by the tangential friction force acting per unit area divided by the velocity gradient under conditions of streamline flow.

An important rheological measurement that is closely related to the resistance to flow is called viscosity.

The Viscosity of a fluid described as the resistance of liquid. For example liquids like water, alcohol, petrol and more flow freely and faster than glycerin solution, honey, and oil. This is because of its physical property and is called viscosity. In simple words, it explains the fluid's flow resistance.

The viscosity is calculated in terms of the coefficient of viscosity. It is constant for a liquid and depends on its liquid's nature. The Poiseuille's method is formally used to estimate the coefficient of viscosity, in which the liquid flows through a tube at the different level of pressures.

The coefficient of viscosity of fluids will be decreased as the temperature increases, while it is inverse in the case of gases. While the coefficient of viscosity of gases will increase with the increase in temperature. The increase in temperature for the fluid deliberate the bonds between molecules. These bonds are directly associated with the viscosity and finally, the coefficient is decreased.

An important question most people do not ask is, why even measure Viscosity? The main reason behind this being, that viscosity data on a material gives manufacturers a chance to predict how the material will react and behave when consumers interact with it.

The most simplistic formula for measuring viscosity is:

Viscosity=shear stress / shear rate

The Coefficient of Viscosity Formula

The force of friction between two layers of fluid having the area in square centimeter and separated by distance will have a velocity is given by:

$F \propto A\, dV/dx$

Or

$f = \eta A\, dV/dx$

Here,

η is coefficient of viscosity

dVdx is velocity gradient

SI Unit of Coefficient of Viscosity

Every liquid has its specific viscosity and the measure of this attribute is called the coefficient of viscosity.

The coefficient of viscosity η is defined as the tangential force F required to maintain a unit velocity gradient between two parallel layers of liquid of unit area A.

The SI unit of η is Newton-second per square meter ($Ns.\ m^{-2}$) or

Pascal-seconds (Pa .s)

Hence the coefficient of viscosity is a measure of the resistance of the fluid to deformation at a given rate due to internal friction.

Unit of Coefficient of Viscosity

The centimetre-gram-second or CGS unit of coefficient of viscosity, η is

dyne-sec/ cm^2 which is equal to Poise.

Where one poise is exactly 0.1 Pa·s.

The meter-kilogram-second or MKS unit is: Kilogram per meter per second or $Kg\ m^{-1}\ s^{-1}$.

Coefficient of Viscosity Unit and Dimension

Since, the formula for coefficient of viscosity is given by,

$\eta = F . d / A . v = MLT^{-2} . L / L^{2} . LT^{-1}$

❖ Steady and Turbulent flow :

Liquids and gases are fluids, as they are characterized by having no fixed shape and flowing easily. We encounter fluids often in nature and everyday life, so we might already be familiar with the difference between steady and turbulent flow, for example, from experiencing turbulence in an airplane. When an aircraft travels through a patch of turbulent air, it gets bumped around and experiences rapid changes in motion. Turbulence is characterized by this kind of chaotic fluid motion that is rapidly changing in direction or speed.

Fluids are able to move at high speeds without being turbulent, such as in a steadily flowing river. Steady flow is characterized by the fluid particles all traveling with similar speed and direction, making for a calm motion that we call steady flow.

The terms "steady" and "turbulent" describe opposite types of flow, and we can characterize flow along a spectrum depending on the change in speed and direction of the fluid.

We can say that a fluid is flowing steadily if its direction and speed change less, and by contrast, a flow is turbulent if the fluid changes in direction and speed more.

The image below illustrates the contrast between steady and turbulent flow. The water starts as a fairly steady flow, but it mixes up after it drops down a ledge. The flow at the bottom of the ledge is far more turbulent since the water is experiencing rapid changes in speed and direction.

Fluids can be difficult to model quantitatively because they are made of vast numbers of moving particles, but there are more qualitative methods of describing patterns of fluid flow. We will model a flowing fluid as consisting of individual streams, or layers, in which the fluid may flow at different speeds. We will use these flow lines, or streamlines, to illustrate how a fluid flows, with the lines representing the direction of the fluid's flow. From this information, we can make some other associations about the flow, as we will explore next.

❖ Reynold's Number:

Reynolds number is a dimensionless quantity that is used to determine the type of flow pattern as laminar or turbulent while flowing through a pipe. Reynolds number is defined by the ratio of inertial forces to that of viscous forces.

It is given by the following relation:

Reynolds Number = Inertial Force / Viscous Force

$$\mathbf{R_e = \rho\ V\ D / \mu}$$

Where,

R_e is the Reynolds number

ρ is the density of the fluid

V is the velocity of flow

D is the pipe diameter

μ is the viscosity of the fluid

If the Reynolds number calculated is high (greater than 2000), then the flow through the pipe is said to be turbulent. If Reynolds number is low (less than 2000), the flow is said to be laminar. Numerically, these are acceptable values, although in general the laminar and turbulent flows are classified according to a range. Laminar flow falls below Reynolds number of 1100 and turbulent falls in a range greater than 2200.

Laminar flow is the type of flow in which the fluid travels smoothly in regular paths. Conversely, turbulent flow isn't smooth and follows an irregular math with lots of mixing.

The significance of Reynolds number

i. The Reynolds number (Re) helps predict flow patterns in different fluid flow situations.

ii. Types of flow are classified into 2 types: laminar flow and turbulent flow.
iii. Reynolds number helps us to determine whether the flow is laminar or turbulent.
iv. At low Reynolds numbers, flows tend to be dominated by laminar (sheet-like) flow
v. At high Reynolds numbers, the flow tends to be turbulent.
vi. Reynolds number is defined by the ratio of inertial forces to that of viscous forces.

❖ Equation of Continuity:

The continuity equation is defined as the product of cross-sectional area of the pipe and the velocity of the fluid at any given point along the pipe is constant.

Continuity equation represents that the product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is always constant. This product is equal to the volume flow per second or simply the flow rate. The continuity equation is given as:

$$R = A\, v = \text{constant}$$

Where,

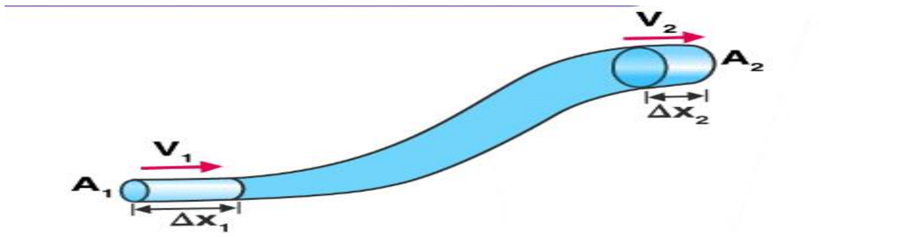
- R is the volume flow rate
- A is the flow area
- v is the flow velocity

Assumption of Continuity Equation

Following are the assumptions of continuity equation:

- The tube is having a single entry and single exit
- The fluid flowing in the tube is non-viscous
- The flow is incompressible
- The fluid flow is steady

Consider,



Now, consider the fluid flows for a short interval of time in the tube. So, assume that short interval of time as Δt. In this time, the fluid will cover a distance of Δx_1 with a velocity v_1 at the lower end of the pipe.

At this time, the distance covered by the fluid will be:

$$\Delta x_1 = v_1 \Delta t$$

Now, at the lower end of the pipe, the volume of the fluid that will flow into the pipe will be:

$$V = A_1 \Delta x_1 = A_1 v_1 \Delta t$$

It is known that ***mass (m) = Density (ρ)*** × ***Volume (V).*** So, the mass of the fluid in Δx_1 region will be:

$$\Delta m_1 = \text{Density} \times \text{Volume}$$

$$\Rightarrow \Delta m_1 = \rho_1 A_1 v_1 \Delta t \text{ ——— (Equation 1)}$$

Now, the mass flux has to be calculated at the lower end. Mass flux is simply defined as the mass of the fluid per unit time passing through any cross-sectional area. For the lower end with cross-sectional area A_1, mass flux will be:

$$\Delta m_1/\Delta t = \rho_1 A_1 v_1 \text{ ——— (Equation 2)}$$

Similarly, the mass flux at the upper end will be:

$$\Delta m_2/\Delta t = \rho_2 A_2 v_2 \text{ ——— (Equation 3)}$$

Here, v_2 is the velocity of the fluid through the upper end of the pipe i.e. through Δx_2 , in Δt time and A_2, is the cross-sectional area of the upper end.

In this, the density of the fluid between the lower end of the pipe and the upper end of the pipe remains the same with time as the flow is steady. So, the mass flux at the lower end of the pipe is equal to the mass flux at the upper end of the pipe i.e. ***Equation 2 = Equation 3.***

Thus,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \text{ ——— (Equation 4)}$$

This can be written in a more general form as:

ρ A v = constant

The equation proves the law of conservation of mass in fluid dynamics. Also, if the fluid is incompressible, the density will remain constant for steady flow. So, $\rho_1 = \rho_2$.

Thus, ***Equation 4*** can be now written as:

$A_1 v_1 = A_2 v_2$

This equation can be written in general form as:

A v = constant

Now, if ***R*** is the volume flow rate, the above equation can be expressed as:

R = A v = constant

This was the derivation of continuity equation.

- **Energy of Flowing liquid:**
  A liquid in motion possesses pressure energy, kinetic energy and potential energy.

  (i) Pressure energy

  It is the energy possessed by a liquid by virtue of its pressure.

  Consider a liquid of density ρ contained in a wide tank T having a side tube near the bottom of the tank as shown in Fig. . A frictionless piston of cross sectional area 'a' is fitted to the side tube. Pressure exerted by the liquid on the piston is $P = h \rho g$ where h is the height of liquid column above the axis of the side tube.

  If x is the distance through which the piston is pushed inwards, then

  Volume of liquid pushed into the tank = ax

  ∴ Mass of the liquid pushed into the tank = ax ρ
  As the tank is wide enough and a very small amount of liquid is pushed inside the tank, the height h and hence the pressure P may be considered as constant.

Work done in pushing the piston through the distance x = Force on the piston x distance moved

(i.e) W = Pax

This work done is the pressure energy of the liquid of mass axρ.

∴ Pressure energy per unit mass of the liquid = Pax /axp = P/p

(ii) Kinetic energy

It is the energy possessed by a liquid by virtue of its motion.

If m is the mass of the liquid moving with a velocity v, the kinetic energy of the liquid = $1/2\ mv^2$

Kinetic energy per unit mass = $1/2mv^2 \ / \ m = v^2 / 2$

(iii) Potential energy

It is the energy possessed by a liquid by virtue of its height above the ground level.

If *m* is the mass of the liquid at a height *h* from the ground level, the potential energy of the liquid = *mgh*

Potential energy per unit mass = mgh/ m = gh

Total energy of the liquid in motion = pressure energy + kinetic energy + potential energy.

- **Bernoulli's Principle :**

Bernoulli's principle formulated by Daniel Bernoulli states that as the speed of a moving fluid increases (liquid or gas), the pressure within the fluid decreases. Although Bernoulli deduced the law, it was Leonhard Euler who derived Bernoulli's equation in its usual form in the year 1752.

It state that,

'The total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure and the kinetic energy of the fluid motion, remains constant'.

Bernoulli's principle can be derived from the principle of conservation of energy.

Bernoulli's equation formula is a relation between pressure, kinetic energy, and gravitational potential energy of a fluid in a container.

The formula for Bernoulli's principle is given as:

$$p + 1/2\ \rho\ v^2 + \rho gh = \text{constant}$$

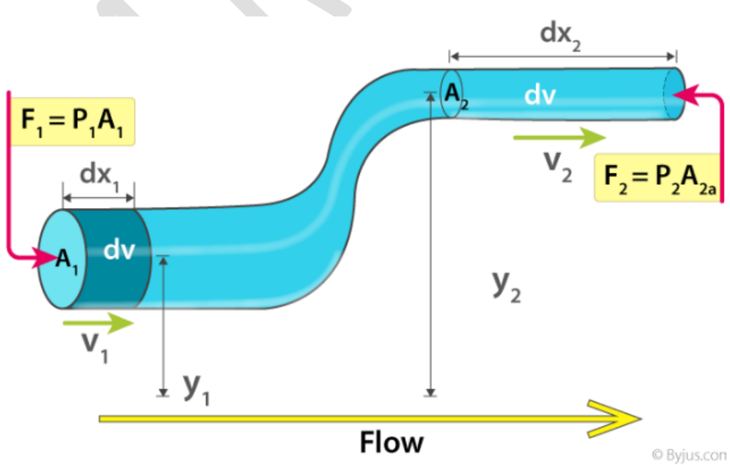
Where,

- p is the pressure exerted by the fluid
- v is the velocity of the fluid
- ρ is the density of the fluid
- h is the height of the container

Bernoulli's equation gives great insight into the balance between pressure, velocity and elevation

Bernoulli's Equation Derivation

Consider a pipe with varying diameter and height through which an incompressible fluid is flowing. The relationship between the areas of cross-sections A, the flow speed v, height from the ground y, and pressure p at two different points 1 and 2 is given in the figure below.



Assumptions:

- The density of the incompressible fluid remains constant at both points.

- The energy of the fluid is conserved as there are no viscous forces in the fluid.

Therefore, the work done on the fluid is given as:

$dW = F_1dx_1 – F_2dx_2$

$dW = p_1A_1dx_1 – p_2A_2dx_2$

$dW = p_1dV – p_2dV = (p_1 – p_2)dV$

We know that the work done on the fluid was due to conservation of gravitational force and change in kinetic energy. The change in kinetic energy of the fluid is given as:

$dK=1/2\ m_2v_2^2-1/2m_1v_1^2=1/2\rho dV(v_2^2-v_1^2)$

The change in potential energy is given as:

$dU = mgy_2 – mgy_1 = \rho dVg(y_2 – y_1)$

Therefore, the energy equation is given as:

$dW = dK + dU$

$(p_1 – p_2)dV = 1/2\rho dV(v_2^2-v_1^2) + \rho dVg(y_2 – y_1)$
$(p_1 – p_2) = 1/2\rho(v_2^2-v_1^2) + \rho g(y_2 – y_1)$
Rearranging the above equation, we get

$p_1+1/2\rho v_1^2+\rho gy_1=p_2+1/2\rho v_2^2+\rho gy_2$

This is Bernoulli's equation.

- **Applications of Bernoulli's Principle:**

1) Venturimeter:

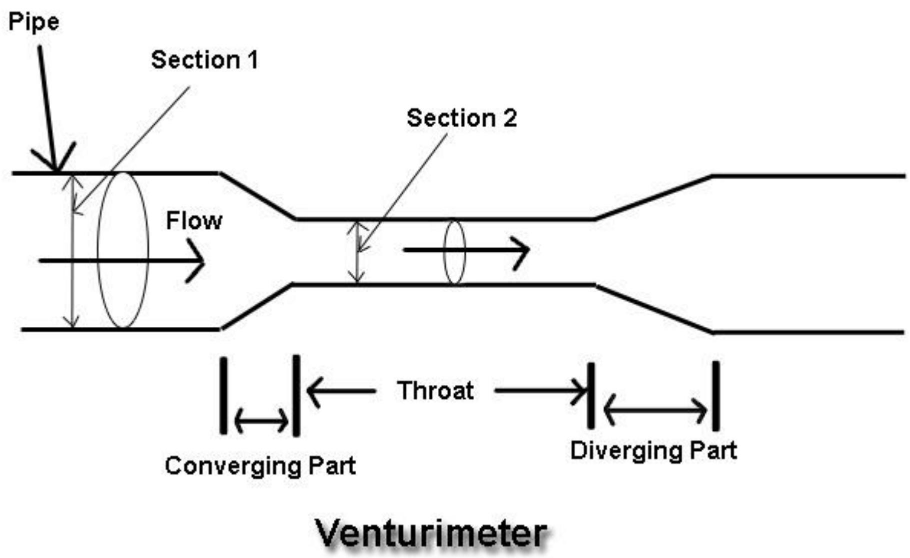
Venturimeter is a device that is used to measure the rate of flow of fluid through a pipe. This device is based on the principle of Bernoulli's Equation. Venturimeter is named after G.B Venturi who developed the principle of venturimeter in 1797 but this principle comes into consideration with the help of C. Herschel who developed the first venturimeterin 1887.

Main parts of venturimeter :-

1. Covering part : It is the part of the venturimeter where the fluid converges .

2. Throat : It is the portion that lies in between the converging and diverging part of venturi. In the throat portion the cross section is much less than converging and diverging portion. When the reaches the throat, its velocity increases and pressure decreases.

3. Diverging part : It is the part of the venturimeter where the fluid gets diverges and the cross-section area increases.



Construction :

As stated above it has three parts converging part, throat and diverging part. These three parts are arranged in systematic order.

First one is **inlet section or converging section.** It is the region where the cross section emerges into conical shape for the connectivity with the throat region. . In this part cross section area decreases from beginning to ending. This section is connected to inlet pipe on one end and cylindrical throat on the other end. The angle of convergence is generally 20-22 degrees .

Second one is **cylindrical throat .**It is the middle part of the venturimeter. It is the cylindrical pipe in venturimeter through which the fluid passes after converging in the convergent section. Throat has generally a diameter of throat is half the diameter of pipe. The diameter of the throat remains same through out its length.

Last one is **diverging section .** It is the end of the venturimeter. On one side it is attached to throat of venturimeter and on the other side it is attached to the pipe. The divergent section has an angle 5 to 15 degrees . The diverging angle is less than the converging angle because the length of the diverging cone is larger than converging cone. The main reason of the small diverging angle is to avoid flow seperation from the walls.

Working :-

Venturimeter works on the principle of Bernoulli's equation i.e when velocity increases pressure decreases . Cross section of throat is less than cross section of inlet pipe. Since the cross -section decreases from inlet pipe to throat, the velocity of the fluid increases and hence the pressure decreases. Due to decrease in pressure, a pressure difference is created between the inlet pipe and throat of the venturimeter . This pressure difference can be measured by placing a differential manometer between the inlet section and throat section or by using two guages at inlet section and throat. After getting the pressure difference flow rate through pipe is calculated.

Expression for the rate of flow through Venturimeter

Consider a venturimeter is fitted in a horizontal pipe through which fluid (water) is flowing as shown in figure below.

Let a1 = cross-section area of inlet pipe
d1= diameter of inlet pipe
v1 = velocity at inlet pipe
p1 = pressure at inlet pipe

also,
a2 = cross-section area of throat
d2= diameter of throat
v2 = velocity at throat
p2 = pressure at inlet throat

Applying bernoull's equation at section (1) and (2) , we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As the pipe is horizontal, so $Z_1 = Z_2$
Hence we get,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

$$\frac{p_1 - p_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g} \quad \ldots\ldots\ldots (1)$$

$(P_1 - P_2)/\ \rho g$ is the difference of pressure heads at section 1 and 2 and is equal to h. So eq (1) becomes

$$h = \frac{v_2^2 - v_1^2}{2g} \ldots\ldots..(2)$$

Now applying continuity at section (1) and (2), we get

$$a_1 v_1 = a_2 v_2$$

$$v_1 = \frac{a_2 v_2}{a_1}$$

Placing the value of v1 in eq (2) and solving, we get

$$v_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

Now , Rate of flow of fluid or Discharge (Q) can be stated as

$$Q = a_2 v_2$$

Substituting value of v2 in this equation, we get

$$Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

Q is the the theoretical discharge under ideal conditions. Actual discharge will be less than this. The actual discharge is given by,

$$Q_{act} = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

Where C_d is coefficient of venturimeter and its value is always less than 1.

2) Pitot tube :

A pitot tube can be used to measure fluid flow velocity by converting the kinetic energy in a fluid flow to potential energy.

he principle is based on the Bernoulli Equation where each term of the equation can be interpreted as pressure

p + 1/2 ρ v² + ρ g h

= p + 1/2 ρ v² + γ h

= constant along a streamline (1)

where

p = static pressure (relative to the moving fluid) (Pa)

ρ = density of fluid (kg/m³)

v = flow velocity (m/s)

γ = ρ g = specific weight (N/m³)

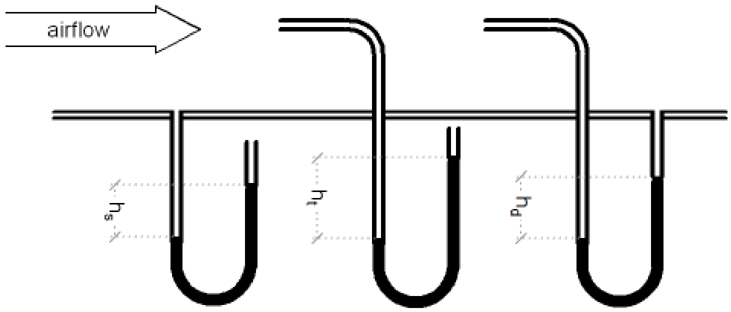
g = acceleration of gravity (m/s²)

h = elevation height (m)

Each term of the equation has the dimension force per unit area N/m^2 *(Pa)* - or in imperial units lb/ft^2 *(psi)*.

The pitot tube is a simple and convenient instrument to measure the difference between **static, total** and **dynamic pressure (or head)**.

The head - *Δh* - (or pressure difference - *Δp*) can be measured and calculated with the help of u-tube manometers, electronic pressure transmitters or similar instrumentation.



❖ Applications of Viscous fluids:

The importance of viscosity can be understood from the following examples –

(i) The knowledge of the coefficient of viscosity of organic liquids is used to determine their molecular weights.

(ii) The knowledge of the coefficient of viscosity and its variation with temperature helps us to choose a suitable lubricant for specific machines. In light machinery thin oils (for example, lubricant oil used in clocks) with low viscosity is used. In heavy machinery, highly viscous oils (example: grease) are used. In lubrication, viscosity is the most significant characteristic of lubricating oils and, what is often forgotten, it is also very vital in greases. Viscosity is the resistance to movement. Water flows fast so it has low viscosity, honey has a high viscosity.

(iii) The viscosity of a few medications, such as the various liquids used to remove warts, has also been modified for easier application. Drug companies manufacture medicines, such as cough syrup, that have a high viscosity yet are still drinkable, in order to coat the throat.

(iv) The viscosity of paints, varnishes, and comparable household products is closely regulated so that the paints and varnishes can be applied smoothly and evenly with a brush roller.

(v) Viscosity plays a considerable function in the preparation and serving of food. Cooking oils may or may not change viscosity as they heat, while many become much more viscous as they cool. Fats, which are reasonably viscous when heated, become solid when chilled.

(vi) Manufacturing equipment requires suitable lubrication to run effortlessly. Lubricants that are too viscous can jam and clog pipelines. Lubricants that are too thin present too little protection for moving parts.

(vii) The viscosity of coatings is one of the key parameters that determine the achievement of the coating procedure. Because the repeatability and reproducibility of the coating procedure are often closely linked to the coating's viscosity, it is a key parameter to control.

Real- life applications of liquid with low viscosity:

1. One example is brake fluid. Brake fluid transmits force through the braking system, and it would not operate properly if it had a different viscosity,
2. Gum that we use are highly viscous, to hold mating part before powerfully before sticking action complete,
3. Lubricants for machine systems also work the way they do in part as a result of their viscosity,
4. using coolant while machining at lathe shops,
5. petrol (as a cleaner) being used in mobile shops.

Viscosity measurements of cosmetics are often performed for shelf-life testing to find out constancy. Viscosity measurements are a significant instrument for a receptive determine of material changes. Many of the formulations are complex emulsions with structures that vary over time and may be reflected in viscosity changes