K.T.S.P. Mandal's

Hutatma Rajguru Mahavidyalaya

Rajgurnagar-410505

Third Year B.Science

V– Sem (2019 CBCS Pattern)

As per the new syllabus

Subject- Classical Mechanics

Mr. N.D.Barne

Department of Physics

nileshbarne11@gmail.com

Topic : Lagarangian and Hamiltonian Formulation

Lagarangian and Hamiltonian Formulation

❖ Limitations of Newton's Laws of Motion:

Limitations of Newtonian Mechanics Newtonian mechanics can be used to describe many every-day macroscopic phenomena. When we study macroscopic phenomena, we can measure both the position and the linear momentum of objects of interest with great precision.

However, when we start to study microscopic object we discover that we can no longer measure the position and the linear momentum with great precision. In fact, the actual measurement may influence the state of the system. In this regime, our capability of determining the position and the linear momentum of an object are limited by the Heisenberg uncertainty principle, which states that If we measure the position with infinite precision, the uncertainty in the linear momentum approaches infinity.

In this regime, Newtonian mechanics can no longer be used, and we need quantum mechanics to describe microscopic systems.

The limitations of Newtonian mechanics also appear when we study motion with velocities close to the speed of light. In this regime, we need the theory of relativity.

One fundamental assumption in the theory of relativity is that the speed of light is constant, the same in each reference frame. This is clearly inconsistent with Newtonian mechanics, and the rules that govern transformations of position and velocity between coordinate systems.

Another limitation of Newtonian mechanics becomes obvious when we try to describe systems with large numbers of particles. Even if we know all of the details of the interaction between the particles, is becomes very difficult to predict the properties of the system by carrying out calculations involving the each individual interaction between all the particles. Such systems can be described by theory of statistical mechanics, which relates the properties of microscopic interactions to the average macroscopic properties of the system.

❖ Constraints:

If the motion of a particle or system is restricted by one or more conditions, then the number of independent ways to move the particle freely is reduced. "The limitations or restrictions on the motion of the system are called constraints and this type of motion of system having constraint is called constraint motion.

Types of Constraints:

- A constrained motion is a motion which cannot proceed arbitrarily in any manner.
- Particle motion can be restricted to occur (1) along with some specified path (2) on the surface (plane or curved) arbitrarily oriented in space.

- Imposing constraints on a mechanical system is done to simplify the mathematical description of the system.
- Constraints expressed in the form of equation $f(x_1,y_1,z_1,\ldots\ldots,x_n,y_n,z_n:t)=0$f(x1,y1,z1,……,xn,yn,zn:t)=0 are called **holonomic constraints**.
- Constraints not expressed in this fashion are called **non-holonomic constraints**.
- **Scleronomic constraints** are independent of time.
- Constraints containing time explicitly are called **rheonomic**.

❖ Degree of Freedom:

Degree of freedom is any of the number of independent quantities necessary to express the values of all the variable properties of a system .A system composed of a point moving without constraints are needed to determine in space. Consider a motion of a particle. These three co-ordinates can be (x,y,z). The particle is free to move along any one axis independently with change in one co-ordinate only. Thus, the particle has three degrees of freedom.

❖ Generalized co-ordinates:

An important characteristic of any mechanical system is the number of degrees of freedom. The number of degrees of freedom is the number of coordinates needed to specify the location of the objects. Therefore, if there are N free objects, there are 3N degrees of freedom. But if there are constraints on the objects, then each constraint removes one degree of freedom. The total number of degrees of freedom for a system of N objects and n constraints is 3N − n. As an example, consider 3 free objects. This system has a total of 9 degrees of freedom. If we constrain the separation between the three to be fixed, we loose three degrees of freedom and therefore our system has only 9 − 3 = 6 degrees of freedom. These 6 degrees of freedom could be selected using any convenient set, for example the 3 coordinates of the center of mass plus the 3 Euler angles. Note, the number of degrees of freedom is independent of the set of coordinates selected. From the above discussion, we are free to select the set of coordinates used to describe a given system as long as the number of coordinates minus the number of constraints gives the number of degrees of freedom for the system.

Note that the number of coordinates and number of constraints does not have to be the same for all possible choices, as illustrated in the 3 mass example 9 coordiantes to 6 coordinates 3 constraints to zero constraints, but the difference must be constant.

To describe a system, we can use any set of parameters that unambiguously represent it. These parameters do not need to have dimensions of length. They are referred to as generalized coordinates. The best choice of generalized coordinates for any given system is one that takes advantage of any symmetries it may have and one where the coordinates are independent. Coordinates that are tied to the symmetry of the problem, lead to conserved momenta. If the coordinates can be varied independently without violating the constraints, their number is equal to the number of degrees of freedom. To simplify most problems, one starts by writing the dynamical equations in rectangular coordinates. Then one transforms to the generalized coordinates. The transformation equations between the two sets is given formally by the following transformation equations

$$x_1 = x_1(q_1, q_2, \ldots, q_{3N}) \qquad (1)$$

$$y_1 = y_1(q_1, q_2, \ldots, q_{3N})$$

$$z_1 = z_1(q_1, q_2, \ldots, q_{3N})$$
$$\vdots$$
$$x_{3N} = x_{3N}(q_1, q_2, \ldots, q_{3N})$$
$$y_{3N} = y_{3N}(q_1, q_2, \ldots, q_{3N})$$
$$z_{3N} = z_{3N}(q_1, q_2, \ldots, q_{3N}) \qquad (2)$$

The constraints are selected at a fixed time, and they are required to be continuous, differentiable, and single valued. The q_i describe the configuration of the system. If they are independent of each other, they form a 3N dimensional space called configuration space. The properties of the configuration space are given by the differential displacements of the sets of coordinates

$$dx_i = \sum_j \partial xi / \partial qj \times dqj \qquad (3)$$

- **Configuration space**

Mechanics concerns itself with dynamical systems – systems whose configuration evolves in time according to some deterministic law. To define a dynamical system, then, one first needs to specify what one means by "configuration". We need to define a configuration space for the system. This is a set of variables which characterizes what the system is doing at a given time — it is the set of quantities which define the system we are studying. Normally, the configuration space is a continuous space (a "manifold"). The dimension of the configuration space is then called the number of degrees of freedom of the system.

We shall see that a better characterization of what a system is doing at any given time is given by the phase space of the system. There are various versions of the phase space. For now you can think of it as the space of possible initial conditions for the system, i.e., the set of conditions you need to uniquely specify a solution to the equations of motion.

Example: Point particles

In many ways the simplest dynamical system is that of a "point particle", which can represent any number of physical systems whenever the internal structure of the system can be neglected for the purposes at hand. Thus, when calculating the forces on a car going around a curve, one can often idealize the car as a mathematical point located at the center of mass. A good first pass at the motion of the earth-sun system consists of assuming that these bodies are represented by point masses located at their respective centers of mass. A point particle can be viewed as the discrete building block in a continuum of matter, etc.

- Principle of Virtual of Work done :
  Consider a system of N particles under time dependent holonomic constraints. If $q_1, \ldots, q_s$ be a set of generalized coordinates, the virtual displacement of the i-th particle is given

$$dri = \sum_{j=1}^{s} \quad \partial ri\ \partial qj\ \delta q$$

Suppose the system, under the action of applied forces as well as those of constraints, is in equilibrium. The total force acting on each particle is then zero,

$$F_i = 0; \; (i = 1, \ldots, N)$$

We then have, for the virtual work done by F_i in the displacement δr_i is

$$F_i \cdot \delta r_i = 0$$

so that the total work done is $\delta W = \sum_i F_i \cdot \delta r_i = 0$

The total force acting on any particle can be split into two: an applied part and a part due to the constraints, $F_i = F^a{}_i + F^c$

then we have

$$\delta W = \sum_i F_i \cdot \delta r_i + \sum_i F^c{}_i \cdot \delta r_i = 0$$

The forces of constraints (e.g. normal reaction, tension, rigid body constraints etc.) do not do any work. This is general true of scleronomic holonomic constraints and this statement is central hypothesis in the principle of virtual work.

- D' Alembert's Principle of Virtual Work:

The D' Alembert's principle, developed from an idea originally due to Bernoulli, is to use the fact that according to Newton's law, the force applied on a particle results in a rate of change of its momentum,

$$\mathbf{F}_i = \dot{\mathbf{p}}_i$$

$\dot{p}_i$ is known as the inertial force or pseudo force acting on the particle. One can then think of bringing the body to equilibrium by applying a pseudo force $-\dot{p}_i$ on the i-th particle of the system $(1 \le i \le N)$,

$$F_i - \dot{p}_i = 0$$

Note that F_i contains both applied and constraint forces acting on the i-th particle. Once again, we can split F_i into two parts and write the above equation for virtual displacements as

$$\sum_i (F^a{}_i + F^c{}_i - \dot{p}_i) \cdot \delta r_i = 0$$

Since the force of constraints do not do any work, we get

$$\sum_i (F^a{}_i - \dot{p}_i) \cdot \delta r_i = 0$$

Above equation is known as D' Alembert's Principle of Virtual Work.