* **Discrete uniform distribution.**

Let X be discrete random variable. taking values 1, 2, ... n X is said to follow a discrete uniform distribution if it p.m.f given by

$$P(X = x) = \frac{1}{n} \quad ; \quad X = 1, 2 \ldots\ldots n$$
$$= 0 \quad , \quad \text{otherwise.}$$

Let X denote the number of on the face of an unbiased die when it is rolled

⟹ $\Omega = \{1, 2, 3, 4, 5, 6\}$.

$$P(X) = \frac{1}{6} \qquad X = 1, 2, 3, 4, 5, 6.$$
$$= 0$$

A computer generate a digit randomly from 0 to 9

$\Omega = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$P(x) = \frac{1}{10} \qquad X = 0 \ldots\ldots 9$$
$$= 0.$$

Mean & Variance :

$$P(X) = \frac{1}{n} \quad ; \quad X = 1 \ldots\ldots n.$$
$$= 0 \qquad \text{otherwise.}$$

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$$\text{Mean} = \mu_1' = E(x) \sum_{i=1}^{n} p(x)$$

$$= \sum_{i=1}^{n} x \frac{1}{n}$$

$$= \frac{1}{n} \sum_{i=1}^{n} x$$

$$= \frac{1}{n} \sum_{i=1}^{n} 1 + 2 + 3 + \dots + n$$

$$= \frac{1}{n} \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2n}$$

$$= \frac{n+1}{2}$$

$$\mu_2 = \text{variance} = Var(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \mu_2' = \sum_{i=1}^{n} x^2 p(x)$$

$$= \frac{1}{n} \sum_{i=1}^{n} x^2$$

$$= \frac{1}{n} (1^2 + 2^2 + 3^2 + \dots + n^2)$$

$$= \frac{\cancel{n}(n+1)(2n+1)}{6\cancel{n}}$$

$$= \frac{(n+1)(2n+1)}{6}$$

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$$E(x^2) = Var(x) = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{2n^2+n+2n+1}{6} - \frac{n^2+2n+1}{4}$$

$$\cancel{= \frac{2n^2+n+2n+1-n^2+2n+1}{6+4}}$$

$$\cancel{= 4n^2+2n+4n+2}$$

$$= \frac{4n^2+2n+4n+2}{12} - \frac{3n^2+6n+1}{12}$$

$$= \frac{4n^2+6n+2-3n^2+6n+1}{12}$$

$$= \frac{n^2-1}{12}$$

$$S.D = \frac{n^2-1}{12}$$

* Bernoulli distribution :-

P.M.F bernoulli distribution

$$P(x) = p^x q^{n-x} \qquad x = 0, 1$$
$$= 0 \qquad , \text{otherwise}.$$

$$p + q = 1$$
$$q = 1 - p.$$

Q. sets of a new born child is recorded in the hospital. Male = 1, female = 0.
∴ expected value is 0, 1

Q. Seeds are germinated : germination of a seed is recorded as a success

Q. Student appear for an examination. The result is pass or fail.

$$\text{mean} = \frac{n+1}{2} \qquad \text{var} = \frac{n^2-1}{12}$$

$X \longrightarrow$ Bernoulli (p)

$$\text{P.M.F} = E(x^2) - [E(x)]^2$$

$$= 0 \qquad \text{otherwise}$$

$$\text{Mean} = E(x) = \sum_{i=0}^{1} x\,p(x)$$

$$= \sum_{i=0}^{1} x\,p^x\,q^{n-x}$$

$$= 0p^0 q^{n-0} + 1p^1 q^{1-1}$$

$$= 0 + p$$

$$= p$$

$$\text{Var}(x) = E(x)^2 - [E(x)]^2$$

$$= \sum_{i=0}^{1} x^2 p(x)$$

$$= \sum_{i=0}^{1} x^2 p^x q^{n-x}$$

$$= 0p^0 q^{n-0} + 1p^1 q^{1-1}$$

$$= 0 + p$$

$$= p$$

$\mu_2 = E(x^2) - [E(x)]^2$
$= p - p^2$
$= p(1-p)$
$= pq.$

* Binomial distribution :-
A discrete random variable X taking value 0, 1, 2 .... n is said to follow a binomial distribution with parameter n and p it's if P.M.F is given by

P.M.F of binomial

$p[X = x] = \frac{n}{x} p^x q^{n-1}$ $\quad X = 0, 1, ..., n$

$= 0$ $\quad 0 < P < 1$

$p + q = 1$

Notation :
$X \sim B(n, p)$
mean $= E(X) = np$
Var $= npq$

If $n = 1$ we get burnoulli distribution.

* Application of binomial distribution :-

1) The random experiment should be burnoulli trial
i.e. It should not result either of the two possible distingh outcome. one of them is

turn as access and other is a failure.

2) The burnoulli trial is perform repeatedly a fix number of time say n.

3) All the trials are independent outcome of a trial in not affected by precessding outcome and does not affect the future outcome.

4) The probability of success in any trial is p and it is consist for each trial.

* Real life of example binomial random variable.

1) No. of defective items in a lot a n items produced by a machine.

2) No. of male birth out of n birth in a hospital.

3) No. of correct answer in multiple choice of 10.

4) No. of seed germinated in row of n planted seeds.

5) No. of rainy days in month.

6) No. of recaptured fish in a sample of n fish.

* Additive property :

Let $X \sim B(n, p)$ and by binoraml n, p and capital X and Y are independent then

$$Z = X + Y \sim B(n_1 + n_2, p).$$

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Mean & Variance :-

$$P(x) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

binomial distribution.

$$X \rightarrow B(n, p)$$

$\therefore$ p.m.f $B(n, p)$ $\quad x = 0, 1, \ldots n$
$\quad = 0 \quad$ otherwise.

$$P(x = x) = {}^nC_x \, p^x q^{n-x}$$

$$E(X) = \mu_1' = \sum_{i=1}^{n} x \, P(x)$$

$$= \sum_{i=1}^{n} x \binom{n}{x} p^x q^{n-x}$$

$$E(x) = \sum_{0}^{n} x \frac{n!}{(n-x)!\, x!} p^x q^{n-x}$$

$$= \sum_{0}^{n} \cancel{x} \frac{n!}{(n-x)!\, \cancel{x}(x-1)!} p^x q^{n-x}$$

$$= \sum_{0}^{n} \frac{n!}{(n-x)!\,(x-1)!}$$

$$= \sum_{0}^{n} \frac{n(n-1)!}{(x-1)!\,(n-x)\,[(n-1)-(x-1)]!}$$

$$p^x q^{n-x} \quad ; x = 0, 1, \ldots n$$

$$= \sum_{1}^{n} \frac{n(n-1)!}{(x-1)!(n)[(n-1)-(x-1)!]} p^{x-1} q^{n-x}$$

$$= \sum_{1}^{n} \frac{(n-1)!}{(x-1)!(n-1)-(x-1)!} p^{x-1} q^{n-x}$$

$x = 1, 2 \ldots n$

$$= \sum_{1}^{n} \binom{n-1}{x-1} p^{x-1} q^{(n-1)-(x-1)}$$

by using binomial expansion.

$$= np(p+q)^{n-1}$$

where $p+q = 1$

$= np$ Mean.

$Var(x) = E(x^2) - [E(x)]^2$

$E[x^2] = \mu_2'$

$= E[x(x-1)] = \sum_{i=0}^{n} x(x-1)\, p(x)$

$\cancel{= E[x'^2}$

$$= \sum_{i=0}^{n} x(x-1)\, {}^nC_x\, p^x\, q^{n-x}$$

$$= \sum_{i=0}^{n} x(x-1) \frac{n!}{(n-x)!\,x!}\, p^x\, q^{n-x} \qquad x = 0, 1, 2 \ldots n$$

$$E[X(X-1)] = \sum_{i=0}^{n} x(x-1) \frac{n!}{(n-x)!\,x(x-1)(x-2)!}\, p^x\, q^{n-x}$$

$$= \sum_{i=0}^{n} \frac{n!}{(n-x)(x-2)!}\, p^x\, q^{n-x}$$

$$\mu_2' = \sum_{i=0}^{n} \frac{n!}{(n-x)!\,(x-2)!} p^{x} q^{n-x}$$

$$= \sum_{i=0}^{n} \frac{n(n-1)(n-2)!}{(n-x)!\,(x-2)!} p^{x} q^{n-x}.$$

$$= n(n-1) \sum_{i=0}^{n} \frac{(n-2)!}{(n-x)!\,(x-2)!} p^{x} q^{n-x}$$

$$= n(n-1)p^2 \sum_{i=2}^{n} \frac{(n-2)!}{((n-2)-(x-2))!\,(x-2)!} p^{x-2} q^{(n-2)-(x-2)}$$

$$= \cancel{n(n-1)p^2} \sum_{i=1}^{2} \binom{n-2}{x-1} p^{x-2} q^{(n-2)-(x-2)}$$

by using binomial expansion.

$$n(n-1)p^2 (p+q)^{n-2}$$

where $p+q = 1$

$$E[x^2] = n(n-1)p^2.$$

$$Var(x) = E[x^2] - E[x]^2$$
$$= n(n-1)p^2 - n^2p^2$$
$$= (n^2-n)p^2 - n^2p^2$$

~~Standard~~

$$= n^2p^2 - np^2 - n^2p^2$$

$$= \cancel{n^2p^2} - np^2 - \cancel{n^2p^2} + np$$

$$= -np^2 + np$$

$$= -np(1-q) + np$$

$$= \cancel{-np} + npq + \cancel{np}$$

$$= npq$$

Standard deviation $= \sqrt{Var(x)}$

$$= \sqrt{npq}$$

* Poisson Distribution.

$$x \sim p(m)$$

p.m.f $= p[X = x] = \dfrac{e^{-m} \; m^{x}}{x!}$, $x = 0, 1, 2 \ldots$

$= 0$, otherwise.

Mean = Variance = m

* Real life situation of poisson distribution :

* Poisson distribution is used in the situation when the event under consideration in a rand event in a short interval time. some real life situation are ~~Some~~

1) No. of twins birth occurring in a hospital during a month.

2) No. of patients suffering from bad reaction due to an injection

3) No. of traffic accident on pune - Mumbai express highway.

4) No. of printing mistake on a page of a book.

5) No. of α ~~particals~~ particules emitted from a radio active source.

6) No. of blood donar of group AB.

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* Geometric distribution :-

Notation :- $X \sim \text{Geometric}(p)$

$X \longrightarrow G(p)$

p.m.f

$P(X=x) = pq^{x}$, $x = 0, 1, 2, \ldots n.$

$p + q = 1$

$q = 1 - q$

$= 0$, otherwise.

$$\text{Mean} = E(X) = \mu_1' = \sum_{i=0}^{n} x\,P(x)$$

$$= \sum_{i=0}^{n} x\,pq^{x}$$

$$= p\sum_{i=0}^{n} x\,q^{x}$$

$$= p\sum_{i=1}^{n} x\,q^{x-1}$$

By using geometric distribution.

$$= p\left[q\left(1 + 2q + 3q^{2} + 4q^{3} + 5q^{4} \ldots + xq^{x-1}\right)\right]$$

$$= pq\left[1 + 2q + 3q^{2} + \ldots\right]$$

$$= pq\left[-q\right]^{-2}$$

$$= pq\,(p)^{-2}$$

$$= \frac{pq}{p^2} \qquad = \frac{q}{p}$$

$$var(X) = \frac{q}{p^2}$$

* Real life situation :-

1) no. of bombs drawed until it hit the target.
2) no. of person to be interview for a post until a suitable candidate is found.
3) no. of attempt ~~of~~ required for successfull launching of a rocket