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F.Y.B.Sc

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As per the New Syllabus

Subject - ST-111: Descriptive Statistics I

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*Chapter 4 – Moments, Skewness and Kurtosis*

# Moments :

## Raw Moments / moments about origin

The $r$th raw moment of a given data set is denoted by $\mu_r'$ and is defined as.

$$\mu_r' = \begin{cases} \dfrac{\sum x_i^r}{n} , & r = 1,2,3,4 \text{ (individual data)} \\ \dfrac{\sum f_i x_i^r}{N} , & r = 1,2,3,4 \text{ (grouped data)} \end{cases}$$

| Moment | individual observation | frequency distribution. |
|---|---|---|
| $\mu_1'$ | $\dfrac{\sum x_i}{n} = \bar{x}$ | $\dfrac{\sum f_i x_i}{N} = \bar{x}$ |
| $\mu_2'$ | $\dfrac{\sum x_i^2}{n}$ | $\dfrac{\sum f_i x_i^2}{N}$ |
| $\mu_3'$ | $\dfrac{\sum x_i^3}{n}$ | $\dfrac{\sum f_i x_i^3}{N}$ |
| $\mu_4'$ | $\dfrac{\sum x_i^4}{n}$ | $\dfrac{\sum f_i x_i^4}{N}$ |

$a = 0$

$a = \bar{x}$

$\dfrac{\sum (x_i - a)^r}{n}$

## Central Moments :

For a given data set the $r$th central moment is denoted by $\mu_r$ and is given by

$$\mu_r = \begin{cases} \sum (x_i - \bar{x})^r / n , & \text{individual observation} \\ \dfrac{\sum f_i (x_i - \bar{x})^r}{N} , & \text{frequency distribution} \end{cases}$$

| Central moment | Individual observation | Frequency distribution |
|---|---|---|
| $\mu_1$ | $\frac{\sum (x_i - \bar{x})}{n} = 0$ | $\frac{\sum f_i (x_i - \bar{x})}{N} = 0$ |
| $\mu_2$ | $\frac{\sum (x_i - \bar{x})^2}{n} = \sigma^2$ | $\frac{\sum f_i (x_i - \bar{x})^2}{N} = \sigma^2$ |
| $\mu_3$ | $\frac{\sum (x_i - \bar{x})^3}{n}$ | $\frac{\sum f_i (x_i - \bar{x})^3}{N}$ |
| $\mu_4$ | $\frac{\sum (x_i - \bar{x})^4}{n}$ | $\frac{\sum f_i (x_i - \bar{x})^4}{N}$ |

## Relation between raw and central moments

i) $\mu_1 = 0$ (always)

ii) $\mu_2 = \mu_2' - (\mu_1')^2 = \sigma^2 =$ Variance

iii) $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

iv) $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

Ex: find first four central moments of the following data set.

1, 3, 4, 5, 7

Solution:

$$\mu_t' = \frac{\sum x_i^t}{n}$$

| $x_i$ | $x_i^2$ | $x_i^3$ | $x_i^4$ |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 3 | 9 | 27 | 81 |
| 4 | 16 | 64 | 256 |
| 5 | 25 | 125 | 625 |
| 7 | 49 | 343 | 2401 |
| $\sum x_i = 20$ | $\sum x_i^2 = 100$ | $\sum x_i^3 = 563$ | $\sum x_i^4 = 3364$ |

$$\mu_1' = \frac{\sum x_i}{n} = \frac{20}{5} = 4$$

$$\mu_2' = \frac{\sum x_i^2}{n} = \frac{100}{5} = 20$$

$$\mu_3' = \frac{\sum x_i^3}{n} = \frac{563}{5} = 112.6$$

$$\mu_4' = \frac{\sum x_i^4}{n} = \frac{3364}{5} = 672.8$$

(further using relation between raw & central moments compute values of central moments)

## Properties of central moments

i) $x_i$ $\qquad u_i = x_i \mp a$

$\mu_x$ : central moment of $x$

$\mu_u$ : $u$

$r$th central moment of $u$ = $r$th central moment of $x$

$$\mu_r(u) = \mu_r(x)$$

Central moments are invariant to the change of origin

| $x_i$ | $u_i = x_i - 3$ |
| --- | --- |
| 2 | -1 |
| 5 | 2 |
| 7 | 4 |
| 9 | 6 |
| 12 | 9 |

Here, if we compute First four central moments of $x$ and $u$ we can verify their values will be same

## (2) Effect of change of scale:

$x_i \qquad u_i = c x_i$

$r$th central moment of $u$ $= c^r \cdot$ $r$th central moment of $x$

$$\mu_r(u) = c^r \mu_r(x)$$

Suppose a data set having the central moments 4, 10, 20, 35 obtain the central moments for i) $u = x - 3$

ii) $v = x + 5$

iii) $w = 2x$

iv) $Y = -3X$

v) $Z = \frac{X}{5}$

Solution: Given that the first four central moments of $X$ are 0, 10, 20 and 35 respectively.

i.e. $\mu_1 = 0$

$\mu_2 = 10$

$\mu_3 = 20$

$\mu_4 = 35$

① $U = X - 3$

$\mu_r(U) = \mu_r(X)$

$\mu_1 = 0$

$\mu_2 = 10$

$\mu_3 = 20$

$\mu_4 = 35$

② $V = X + 5$

$\mu_r(V) = \mu_r(X)$

Central moments of $V$ are

$\mu_1 = 0$, $\mu_2 = 10$, $\mu_3 = 20$, $\mu_4 = 35$

③ $W = 2X$

$$\mu_r(W) = 2^r \mu_r(X)$$

Central moments of $W$ are

$$\mu_1(W) = 2^1 \mu_1(X)$$
$$= 2 \times 0$$
$$= 0$$

$$\mu_2(W) = 2^2 \mu_2(X)$$
$$= 4 \times 10$$
$$\mu_2(W) = 40$$

$$\mu_3(W) = 2^3 \mu_3(X)$$
$$= 8 \times 20$$
$$\mu_3(W) = 160$$

$$\mu_4(W) = 2^4 \mu_3(\alpha)$$
$$= 16 \times 35$$
$$\mu_4(W) = 560$$

iv) $$Y = -3X$$

$$\mu_r(Y) = (-3)^r \mu_r(X)$$

$$\mu_1(Y) = (-3)^1 \mu_1(X)$$
$$= -3 \times 0$$
$$\mu_1(Y) = 0$$

$$\mu_2(Y) = (-3)^2 \mu_2(X)$$
$$= 9 \times 10$$
$$\mu_2(Y) = 90$$

$$\mu_3(Y) = (-3)^3 \mu_3(X)$$
$$= -27 \times 20$$
$$\mu_3(Y) = -540$$

$$\mu_4(Y) = (-3)^4 \mu_4(X)$$
$$= +81 \times 35$$
$$\mu_4(Y) = +2835$$

v) $z = \frac{x}{5}$

$$\mu_r(z) = \left(\frac{1}{5}\right)^r \mu_r(x)$$

$$\mu_1(z) = \left(\frac{1}{5}\right) \mu_1(x)$$

$$= \frac{1}{5} \times 0$$

$$\underline{\mu_1(z) = 0}$$

0
10
24
35

$$\mu_2(z) = \left(\frac{1}{5}\right)^2 \mu_2(x)$$

$$= \frac{1}{25} \times 10$$

$$\underline{\mu_2(z) = 0.4}$$

$$\mu_3(z) = \left(\frac{1}{5}\right)^3 \mu_3(x)$$

$$= \frac{1}{125} \times 24$$

$$\underline{\mu_3(z) = 0.16}$$

$$\mu_4(z) = \left(\frac{1}{5}\right)^4 \times \mu_4(x)$$

$$= \frac{1}{625} \times 35$$

$$\underline{\mu_4(z) = 0.056}$$

$\mu_1 = 0$

ex Given that $\bar{x} = 1$, $\mu_2 = 3$, $\mu_3 = 0$, $\mu_4 = 27$ find the first four raw moments

solution: Given $\bar{x} = 1$, $\mu_2 = 3$, $\mu_3 = 0$, $\mu_4 = 27$

i) $\mu_1' = \bar{x} = 1$

ii) $\mu_2 = \mu_2' - (\mu_1')^2$

$3 = \mu_2' - (1)^2$

$\mu_2' = 3 + 1 = 4$

iii) $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$0 = \mu_3' - 3(4)(1) + 2(1)^3$

$0 = \mu_3' - 12 + 2$

$0 = \mu_3' - 10$

$\mu_3' = 10$

iv) $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$27 = \mu_4' - 4(10 \times 1) + 6(4 \times 1)^? - 3(1)^4$

$27 = \mu_4' - 40 + 24 - 3$

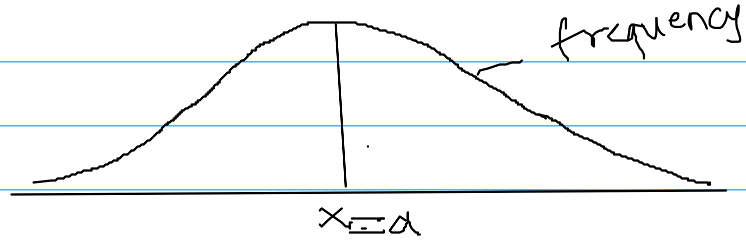
$27 = \mu_4' - 43 + 24$

$27 = \mu_4' - 19$

$\mu_4' = 27 + 19 = 46$

# Skewness and Kurtosis

## Symmetry :




### Properties of Symmetric Distribution:

1) for symmetric distribution
Mean = Median = Mode

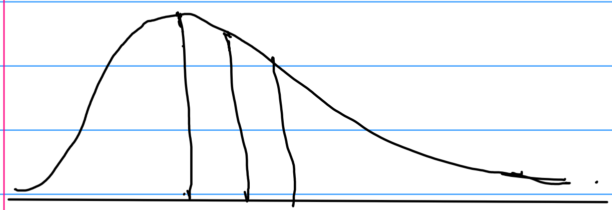
2) $Q_3 - Q_2 = Q_2 - Q_1$

3) For symmetric distribution all odd ordered central moments are zero.

$$\mu_1 = \mu_3 = \mu_5 = \cdots = 0$$

| Class | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 |
|---|---|---|---|---|---|
| frequency | 05 | 10 | 15 | 10 | 05 |

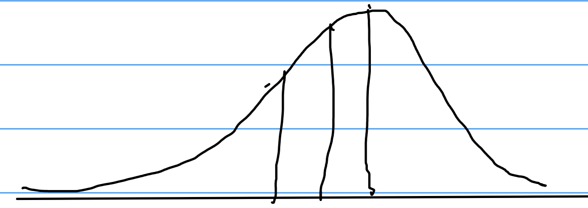
**Skewness**: It is a lack of symmetry.
It is opposite to the symmetry.



$M_o$ $M_e$ $M_n$

positively skewed

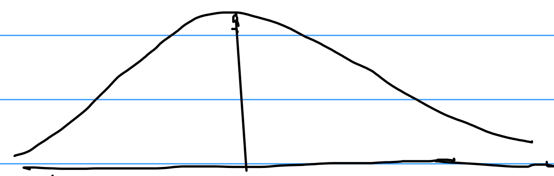
Mean > Median > Mode



$M_n$ $M_e$ $M_o$

Negatively skewed.

Mean < Median < Mode



$M_n = M_e = M_o$

symmetric

$M_n$ = Mean

$M_e$ = Median

$M_o$ = mode

Mean = 12
Median = 11.7
Mode = 8
positive skewness

Mean = 50
Median = 57
Mode = 60
Negative skewness

## Karl pearson's coefficient of skewness (Skp)

$$Skp = \frac{\text{Mean} - \text{mode}}{S.D}$$

### Interpretations of Skp

i) if $Skp > 0$, the distribution is positively skewed

ii) if $Skp < 0$, the distribution is Negatively skewed

iii) if $Skp = 0$, the distribution is symmetric

Ex① For a group of 10 items, $\Sigma x = 452$, $\Sigma x^2 = 24270$ and mode $= 43.7$. Find coefficient of skewness using appropriate formula.?

Solution: Given $\Sigma x = 452$, $\Sigma x^2 = 24270$, $n = 10$

Mode $= 43.7$

$$\text{Mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{452}{10} = \underline{45.2}$$

$$\text{Var}(x) = \frac{\Sigma x^2}{n} - (\bar{x})^2$$

$$= \frac{24270}{10} - (45.2)^2$$

$$= 2427 - 2043.04$$

$$\text{Var}(x) = 383.96$$

$$S.D(x) = \sqrt{383.96} = \underline{19.59}$$

$$Skp = \frac{\text{Mean} - \text{mode}}{S.D}$$

$$= \frac{45.2 - 43.7}{19.59}$$

$$= \frac{1.5}{19.59}$$

$$\underline{Skp = 0.076} \quad \underline{i.e} \; Skp > 0$$

The distribution is positively skewed.

(2) Given that, A.M = 160, mode = 157, $\sigma = 50$ find Karl pearsons coefficient of skewness and interpret the value

Solution:

Given, A.M = 160, mode = 157, $\sigma = 50$

$$SK_p = \frac{\text{Mean} - \text{mode}}{S.D}$$

$$= \frac{160 - 157}{50}$$

$$= \frac{3}{50}$$

$$SK_p = 0.06$$

Here, $SK_p > 0$

The distribution is positively skewed.

Note:

$$SK_p = \frac{\text{mean} - \text{mode}}{SD}$$

In some cases we have given the value of mean & median

$$\text{Mean} - \text{mode} \approx 3(\text{mean} - \text{median})$$

$$SK_p \simeq \frac{3(\text{mean} - \text{median})}{S.D}$$

(3) from the information given below compare the skewness of the two groups

| | Group I | Group II |
|---|---|---|
| Mean | 24 | 22 |
| median | 22 | 25 |
| S.D | 10 | 12 |

solution : we know that,

$$Skp = \frac{mean - mode}{S.D} \simeq \frac{3(mean - median)}{S.D}$$

For Group I

$$Skp \simeq \frac{3(24-22)}{10}$$

$$= \frac{3 \times 2}{10}$$

$$= 6/10$$

$$Skp = 0.6 > 0$$

Group I is positively skewed.

For Group II

$$Skp \simeq \frac{3(mean - median)}{S.D}$$

$$= \frac{3(22-25)}{12}$$

$$= \frac{3 \times -3}{12}$$

$$= -9/12$$

$Sk_p = -0.75 < 0$

The observation from Group II are negatively skewed.

Here we observed that $|Sk_p| = 0.75$ for group II is larger than group I. Hence group II possess more skewness.

(4) A distribution has mean 30, coefficient of variation 20 % and coefficient of skewness 0.3. Find its mode.

Solution: Given: mean $= \bar{x} = 30$

$C.V = 20\%$

$Sk_p = 0.3$

Mode $= ?$

Since,

$$C.V = \frac{S.D}{|\bar{x}|} \times 100$$

$$20 = \frac{\sigma}{30} \times 100$$

$$\sigma = \frac{20 \times 30}{100} = \frac{600}{100}$$

$$\sigma = 6$$

Also,

$$Sk_p = \frac{\text{mean} - \text{mode}}{S.D}$$

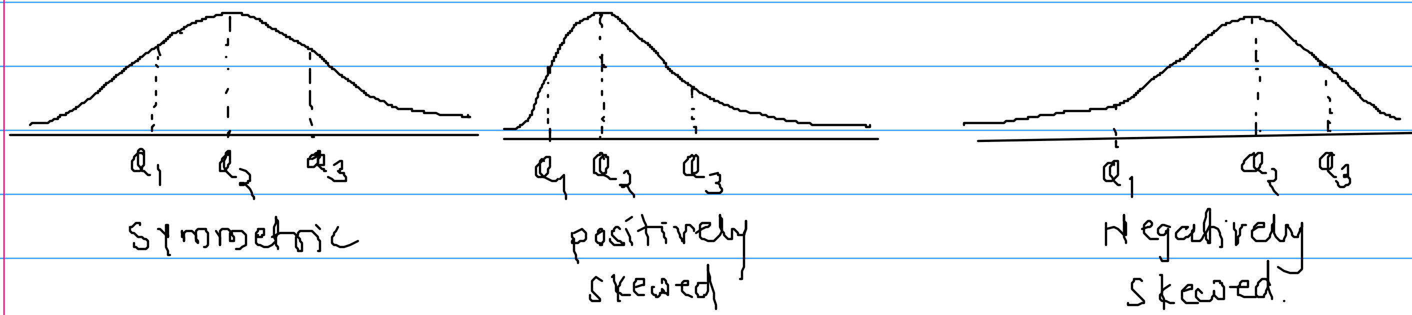
$$0.3 = \frac{30 - \text{mode}}{6}$$

$$0.3 \times 6 = 30 - \text{mode}$$

$$\text{mode} = 30 - 1.8$$

$$\boxed{\text{Mode} = 28.2}$$

## ② Bowley's Coefficient of skewness:




$Q_3 - Q_2 = Q_2 - Q_1$ , $Q_3 - Q_2 > Q_2 - Q_1$ , $Q_3 - Q_2 < Q_2 - Q_1$

$$Sk_b = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$Sk_b = \frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1}$$

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**Interpretation:**

i) If $Sk_b > 0$, then the distribution is positively skewed
ii) If $Sk_b < 0$, then the distribution is Negatively skewed
iii) If $Sk_b = 0$, then the distribution is symmetric

**Result:** The Bowley's coefficient of skewness ($Sk_b$) lies between $-1$ and $1$.

Ex ① Compute Bowley's coefficient of skewness for the following data set.

26, 30, 35, 5, 6, 7, 9, 20, 40, 45, 11, 18, 15, 49, 60

Solution: The given observation in increasing order are

$5, 6, 7, \boxed{9}, 11, 15, 18, \boxed{20}, 26, 30, 35, \boxed{40}, 45, 49, 60$

Here $n = 15$

$Q_K$ = The value of $K\left(\frac{n+1}{4}\right)^{th}$ observation.

$K = 1, 2, 3$

$Q_1$ = The value of $1\left(\frac{n+1}{4}\right)^{th}$ observation

$= \left(\frac{15+1}{4}\right)^{th}$ obsn

$= (4)^{th}$ observation

$Q_1 = 9$

$Q_2$ = The value of $2\left(\frac{n+1}{4}\right)^{th}$ observation

$= 2\left(\frac{15+1}{4}\right)^{th}$ observation

$Q_2$ = The value of $8^{th}$ observation

$Q_2 = 20$

$Q_3 =$ The value of $3\left(\frac{n+1}{4}\right)^{th}$ observation

$= 3\left(\frac{15+1}{4}\right)^{th}$ observation

$=$ The value of $12^{th}$ observation

$Q_3 = 40$

Here we get, $Q_1 = 9$, $Q_2 = 20$, $Q_3 = 40$

$$Sk_b = \frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1}$$

$$= \frac{40 - 2(20) + 9}{40 - 9}$$

$$= \frac{40 - 40 + 9}{31}$$

$$= \frac{9}{31}$$

$Sk_b = 0.2903 > 0$

The distribution is positively skewed.

② Compute the Bowley's coefficient of skewness for the following frequency distribution.

| Marks | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 |
|---|---|---|---|---|---|
| No. of students | 5 | 12 | 32 | 40 | 11 |

Solution:

| | Marks | No. of students ($f_i$) | c.f |
|---|---|---|---|
| | 0-20 | 5 | 5 |
| | 20-40 | 12 | 17 |
| $Q_1$ class ← | 40-60 | 32 | 49 $> N/4$ |
| $Q_2$ class & $Q_3$ class ← | 60-80 | 40 | 89 $> N/2$ $> \frac{3N}{4}$ |
| | 80-100 | 11 | 100 |
| | | $N = \sum f_i = 100$ | |

$$Q_K = L + h\frac{\left(\frac{KN}{4} - c.f\right)}{f}, \quad K = 1,2,3$$

$$Q_1 = L + h\frac{\left(\frac{N}{4} - c.f\right)}{f}, \quad \frac{N}{4} = \frac{100}{4} = 25$$

Here $Q_1$ class is 40-60

$$= 40 + 20\frac{(25-17)}{32}$$

$$= 40 + \left(\frac{20 \times 8}{32}\right)$$

$$= 40 + 5$$

$$Q_1 = 45$$

$$Q_2 = L + h\frac{\left(\frac{N}{2} - c\cdot f\right)}{f}, \quad \frac{N}{2} = \frac{100}{2} = 50$$

$$= 60 + 20\frac{(50 - 49)}{40}$$

$$= 60 + \left(\frac{20 \times 1}{40}\right)$$

$$= 60 + \frac{20}{40}$$

$$= 60 + 0.5$$

$$\underline{Q_2 = 60.5}$$

$$Q_3 = L + h\frac{\left(\frac{3N}{4} - c\cdot f\right)}{f}, \quad \frac{3N}{4} = \frac{3 \times 100}{4} = 75$$

$$= 60 + 20\frac{(75 - 49)}{40}$$

$$= 60 + \left(\frac{20 \times 26}{40}\right)$$

$$= 60 + \left(\frac{26}{2}\right)$$

$$= 60 + 13$$

$$\underline{Q_3 = 73}$$

$$Sk_Q = \frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1}$$

$$= \frac{73 - 2(60.5) + 45}{73 - 45}$$

$$= \frac{118 - 121}{28}$$

$$= \frac{-3}{28}$$

$$Sk_b = -0.1071$$

i.e $Sk_b < 0$

The distribution is negatively skewed.

(5) In a certain frequency distribution the sum of upper and lower quartiles is 45 and the difference between them is 15. If the median is 20, find the coefficient of skewness & interpret the value.

solution:

Given,

$$Q_3 + Q_1 = 45 \quad \text{--- (1)}$$
$$Q_3 - Q_1 = 15 \quad \text{--- (2)}$$
$$\text{Median} = Q_2 = 20$$

by adding eqn (1) & (2)

$$\begin{array}{r} Q_3 + \cancel{Q_1} = 45 \\ + \quad Q_3 - \cancel{Q_1} = 15 \\ \hline 2Q_3 = 60 \end{array}$$

$$Q_3 = 30$$

equation (1) becomes,

$$30 + Q_1 = 45$$
$$Q_1 = 45 - 30$$
$$Q_1 = 15$$

$$Sk_b = \frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1}$$

$$= \frac{30 - 2(20) + 15}{30 - 15}$$

$$= \frac{45 - 40}{15}$$

$$= \frac{5}{15}$$

$$Sk_b = 0.3333$$

$$Sk_b > 0$$

Hence, the distribution is positively skewed.

(4) In a certain frequency distribution upper quartile exceeds the median by 10 units whereas the median exceeds the lower quartile by 7 units. Compute the coefficient of skewness.

Solution:

$$Q_3 - Q_2 = 10$$

$$Q_2 - Q_1 = 7$$

$$Sk_b = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{10 - 7}{10 + 7}$$

$$= \frac{3}{17}$$

$Sk_b = 0.1764$ , $Sk_b > 0$

The distribution is positively skewed.

## (3) Pearsonian coefficient of skewness ($\gamma_1$):

$$\gamma_1 = \sqrt{\beta_1} \quad \text{where, } \beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

i.e $\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\mu_3}{\sqrt{\mu_2^3}}$ , $\mu_2 > 0$

The sign of $\gamma_1$ depends on $\mu_3$ (always)

<u>Interpretation:</u>

i) If $\mu_3 > 0$, $\gamma_1 > 0$ $\Rightarrow$ The distribution is positively skewed

ii) If $\mu_3 < 0$ i.e $\gamma_1 < 0$ $\Rightarrow$ The distribution is Negatively skewed.

iii) If $\mu_3 = 0$ i.e $\gamma_1 = 0$ $\Rightarrow$ The distribution is symmetric.

Ex (1) The first four raw moments of a frequency distribution are 2, 20, 40 & 200 respectively. Comment on the nature of skewness

<u>Solution:</u> Given, $\mu_1' = 2$, $\mu_2' = 20$, $\mu_3' = 40$, $\mu_4' = 200$

$\mu_1 = 0$ (always)

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 20 - (2)^2$

$= 20 - 4$

$$\mu_2 = 16$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$= 40 - 3(20 \times 2) + 2(2)^3$$

$$= 40 - 120 + 16$$

$$= 56 - 120$$

$$\mu_3 = -64$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$= \frac{(-64)^2}{(16)^3}$$

$$= \frac{4096}{4096}$$

$$\beta_1 = 1$$

$$\gamma_1 = \sqrt{\beta_1} = \sqrt{1} = -1 \quad (\text{sine } \mu_3 < 0)$$

$$\gamma_1 = -1 < 0$$

The distribution is negatively skewed.

(2) Given that, $n=100$, $\Sigma x=-10$, $\Sigma x^2=140$, $\Sigma x^3=-40$, $\Sigma x^4=560$

Compute $\beta_1$ and comment on the nature of distribution.

<u>Solution</u>:

$$\mu_r' = \frac{\Sigma x^r}{n}, \quad r=1,2,3,4$$

$$\mu_1' = \frac{\Sigma x}{n} = \frac{-10}{100} = -0.1$$

$$\mu_2' = \frac{\Sigma x^2}{n} = \frac{140}{100} = 1.4$$

$$\mu_3' = \frac{\Sigma x^3}{n} = \frac{-40}{100} = -0.4$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$
$$= 1.4 - (-0.1)^2$$
$$= 1.4 - (0.01)$$
$$\underline{\mu_2 = 1.39}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$
$$= -0.4 - 3(1.4 \times -0.1) + 2(-0.1)^3$$
$$= -0.4 + 0.42 + 2(-0.001)$$
$$= -0.4 + 0.42 - 0.002$$
$$\underline{\mu_3 = 0.018}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$= \frac{(0.018)^2}{(1.39)^3}$$

$$\underline{\beta_1 = 0.0001206}$$

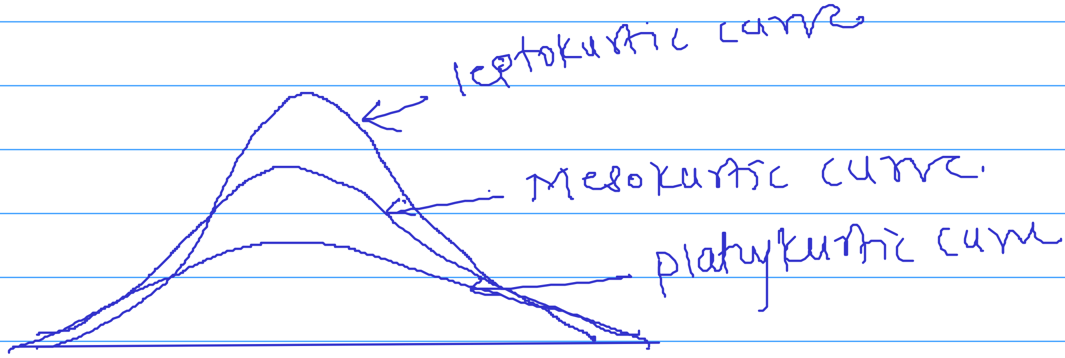
$$\gamma_1 = \sqrt{\beta_1} = 0.0109$$

$$\gamma_1 = 0.0109 > 0 \quad (\mu_3 > 0)$$

The distribution is positively skewed.

## Kurtosis :

Kurtosis is nothing but the property of distribution which expresses its relative peakedness.



## Coefficient of kurtosis ($\gamma_2$)

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\gamma_2 = \beta_2 - 3$$

### Interpretations:

i) If $\gamma_2 > 0$ i.e $\beta_2 > 3$, then the distribution is leptokurtic

ii) If $\gamma_2 < 0$ i.e $\beta_2 < 3$, then the distribution is platykurtic

iii) If $\gamma_2 = 0$ i.e $\beta_2 = 3$, then the distribution is mesokurtic

$\beta_2 = 4.5$ → leptokurtic

$\beta_2 = 2.5$ → platykurtic

$\beta_2 = 3$ → mesokurtic.

Ex ① The first four raw moments of a frequency distribution are 2, 20, 40 & 200 respectively. Comment on the nature of kurtosis.

**Solution:** Given,

$$\mu_1' = 2$$
$$\mu_2' = 20$$
$$\mu_3' = 40$$
$$\mu_4 = 200$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$
$$\gamma_2 = \beta_2 - 3$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$
$$= 20 - (2)^2$$
$$= 20 - 4$$
$$\underline{\mu_2 = 16}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$
$$= 200 - 4 \times 40 \times 2 + 6 \times 20 \times (2)^2 - 3(2)^4$$
$$= 200 - 320 + 480 - 48$$
$$= 680 - 368$$
$$\underline{\mu_4 = 312}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$
$$= \frac{312}{(16)^2}$$
$$= \frac{312}{256}$$
$$\underline{\beta_2 = 1.2187}$$

$$\gamma_2 = \beta_2 - 3 = 1.2187 - 3 = -1.7813$$

$\gamma_2 < 0$, $\Rightarrow$ The distribution is platykurtic.

$\frac{\mu_4}{\mu_2^2} \geq 1 \Rightarrow \mu_4 \geq \mu_2^2$

**Result ①** The pearsonian coefficient, $\beta_2 \geq$

**Result ②** If $\beta_1$ and $\beta_2$ are pearsonian coefficients then,

$$\beta_2 \geq \beta_1 + 1$$

Ex ⑦ Given that $\beta_2 = 2.6$, $\beta_1 = 0.19$, $\mu_2 = 1.2$

Find $\mu_3$ and $\mu_4$

**Solution** :- Here, $\beta_2 = 2.6$, $\beta_1 = 0.19$, $\mu_2 = 1.2$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$0.19 = \frac{\mu_3^2}{(1.2)^3}$$

$$\mu_3^2 = 0.19 \times 1.728 = 0.32832$$

$$\mu_3 = \sqrt{0.32832} = 0.5729$$

$$\boxed{\mu_3 = 0.5729}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$2.6 = \frac{\mu_4}{(1.2)^2}$$

$$\mu_4 = 2.6 \times 1.44 = 3.744$$

$$\boxed{\mu_4 = 3.744}$$

(3) Is the following information consistent?

$\mu_1' = 2, \quad \mu_2' = 20, \quad \mu_3' = 40, \quad \mu_4' = 50$

Justify your answer.

Solution :

$\underline{\mu_1 = 0}$ (always)

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 20 - 2^2$

$= 20 - 4$

$\underline{\mu_2 = 16}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= 40 - 3 \times 20 \times 2 + 2(2)^3$

$= 40 - 120 + 16$

$\underline{\mu_3 = -64}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 50 - 4 \times 40 \times 2 + 6 \times 20(2)^2 - 3(2)^4$

$= 50 - 320 + 480 - 48$

$= 530 - 368$

$\underline{\underline{\mu_4 = 162}}$

we know that $\beta_2 \geq 1$

$\dfrac{\mu_4}{\mu_2^2} \geq 1$

$\mu_4 \geq \mu_2^2$

Here, $\mu_2 = 16 \Rightarrow \mu_2^2 = 256$

we get $\mu_4 = 162 < \mu_2^2$, as $\mu_4 < \mu_2^2$

The given information is inconsistent.