Q4) a) Attempt any one of the following: **[5]**

i) Define Kernal of a linear transformation and if $T : V \to W$ is linear transformation, then prove that Kernal of T is a subspace of V.

ii) Let V is a Finite dimensonal vector space and $T : V \to V$ is linear transformation. Prove that T is injective if and only if $\ker(T) = \{0\}$.

b) Attempt any one of the following: **[5]**

i) Find domain, codomain of $T_2 \circ T_1$ and compute $(T_2 \circ T_1)(x, y)$ if $T_1(x, y) = (2x, -3y, x + y)$, $T_2(x, y, z) = (x, -y, x + y)$.

ii) Find the standard matrix for the linear transfrmation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $T(x, y, z) = (3x - 4y + z,\ x + y - z,\ x + 2y + 3z)$.

Total No. of Questions : 5]

SEAT No. :

PA-2160

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S.Y. B.Sc. (Regular)

MATHEMATICS

MT - 242(A) : Vector Calculus

(2019 CBCS Pattern) (Semester - IV) (24112A)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any Five of the following: **[5]**

a) If $\bar{f}(t)=(t^2+1)\bar{i}+(4t-3)\bar{j}+2t^2\bar{k}$, then find $\lim_{t\to 2}\bar{f}(t)$.

b) If $\bar{r}(t)=\cos 2t\,\bar{i}+3\sin 2t\,\bar{j}$ is the position of a particle in the xy - plane at time 't', then find the velocity vector at t = 0.

c) If $\phi(x,y,z)=xy+yz+xz$, then find $|\nabla\phi|$ at the point (1, 1, 1).

d) Find $\bar{k}$ - component of curl $\bar{\mathrm{F}}$ for the vector field $\bar{\mathrm{F}}=(y\sin x)\bar{i}+(x\sin y)\bar{j}$ on the plane.

e) Give parametrization of the cone $z=\sqrt{x^2+y^2}$, $0\le z\le 1$.

f) Evaluate $\int_c xds$, where c is the curve $x = t, y = t^2$, $0\le t\le 2$.

g) State Divergence Theorem in three dimensions.

P.T.O.

Q2) a) Attempt any one of the following: **[5]**

i) If $\overline{u}(t)$ and $\overline{v}(t)$ are differentiable vector functions of 't' then prove that $\frac{d}{dt}\left[\overline{u}(t)\times\overline{v}(t)\right]=\overline{u}(t)\times\frac{d}{dt}\overline{v}(t)+\frac{d}{dt}\overline{u}(t)\times\overline{v}(t)$.

ii) If $\overline{r}(t)$ is a differentiable vector function of 't' and length of $\overline{r}(t)$ is constant then show that $\overline{r}(t)\cdot\frac{d}{dt}\overline{r}(t)=0$.

b) Attempt any one of the following: **[5]**

i) Find the work done by the force, $\overline{\mathrm{F}}=xy\overline{i}+y\overline{j}-yz\overline{k}$, along a curve, $\overline{r}(t)=t\overline{i}+t^2\overline{j}+t\overline{k}$, in moving an object from the point $t = 0$ to $t = 1$.

ii) Find unit tangent vector $\overline{\mathrm{T}}$, principal normal vector $\overline{\mathrm{N}}$ and curvature k for the curve in space $\overline{r}(t)=3\sin t\,\overline{i}+3\cos t\,\overline{j}+4t\,\overline{k}$.

Q3) a) Attempt any one of the following: **[5]**

i) State Green's Theorem in Normal Form and use it to find the outward flux for the vector field $\overline{\mathrm{F}}=\left(y^2-x^2\right)\overline{i}+\left(y^2+x^2\right)\overline{j}$, where c is the triangle bounded by $y = 0$, $x = 3$ and $y = x$.

ii) Let $\overline{\mathrm{F}}$ be a vector field and C be any closed curve in a region D. Prove that the vector field $\overline{\mathrm{F}}$ is conservative if and only if $\oint_c \overline{\mathrm{F}}\cdot d\overline{r}=0$.

b) Attempt any one of the following: **[5]**

i) Show that the differential form in the integral is exact and evaluate the integral $\int_{(0,0,0)}^{(1,2,3)} 2xy\,dx + \left(x^2 - z^2\right)dy - 2yz\,dz$.

ii) Use parametrization to find the flux of $\overline{\text{F}} = yz\,\overline{i} + x\,\overline{j} - z^2\,\overline{k}$ outward through the parabolic cylinder $y = x^2,\ 0 \le x \le 1,\ 0 \le z \le 4$.

Q4) a) Attempt any one of the following: **[5]**

i) Define surface integral of a scalar function and evaluate $\iint_s y\,d\sigma$, where *s* is the portion of the cylinder $x^2 + y^2 = 3$ that lies between *z* = 0 and *z* = 6.

ii) Define divergence of a vector function $\overline{\text{F}}$ and determine whether the vector function, $\overline{\text{F}} = x^3\,\overline{i} + x^2y\,\overline{j} + x^2z\,\overline{k}$ is solenoidal.

b) Attempt any one of the following: **[5]**

i) Using Stoke's Theorem, evaluate $\iint_s \nabla \times \overline{\text{F}} \cdot \overline{n}\,ds$ for the vector field $\overline{\text{F}} = y\,\overline{i} - x\,\overline{j}$, over the hemisphere, $\text{S}: x^2 + y^2 + z^2 = 9,\ z \ge 0$.

ii) Use the Divergence Theorem to evaluate $\iint_s \overline{\text{F}} \cdot \overline{n}\,d\sigma$, where $\overline{\text{F}} = x^2\,\overline{i} + y^2\,\overline{j} + z^2\,\overline{k}$ over the cube cut from the first octant by the planes *x* = 1, *y* = 1, *z* = 1.

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