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## S.Y.B.Sc. <br> STATISTICS

## ST-242: Sampling Distribution and Exact Tests

 (24172) (2019 CBCS Pattern)(Semester-IV) (Paper -II)Time : 2 Hours]
[Max. Marks : 35
Instructions to the candidates:

1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Use of statistical tables and calculator is allowed.

Q1) Attempt each of the following:
a) In each of the following cases, choose the correct alternative : [1 each]
i) Let $X \rightarrow G(1,3)$ then variance of X is
A) 3
B) $\frac{1}{3}$
C) 1
D) 9
ii) Let $X \rightarrow \chi_{6}{ }^{2}$ then which of the following is true?
A) $\mu_{2}{ }^{\prime}=6$
B) $\mu_{2}^{\prime}=12$
C) $\mu_{2}^{\prime}=36$
D) $\mu_{2}^{\prime}=48$
iii) Let $X \rightarrow t_{7}$ then distribution of $Y=\frac{1}{X^{2}}$ is
A) $F_{1,1}$
B) $F_{1,7}$
C) $F_{7,7}$
D) $F_{7,1}$
b) In each of the following, state whether the given statement is true or false:
i) Let $X \rightarrow N\left(\mu, \sigma^{2}\right) \mu$ known, the test statistic for testing $H_{0}: \sigma^{2}=\sigma_{0}{ }^{2}$ Vs $H_{1}: \sigma^{2} \neq \sigma_{0}{ }^{2}$ follows chi-square distribution.
ii) Let $X \rightarrow t_{10}$ then the mode of X is 10 .

Q2) Attempt any two of the following:
a) Obtain moment generating function of chi square distribution with $n$ degrees of freedom.
b) Derive an expression for $\mathrm{r}^{\text {th }}$ raw moment of F-distribution with $n_{1}$ and $n_{2}$ degrees of freedom. Hence find mean of the distribution.
c) Describe the test procedure for testing $H_{0}: \sigma^{2}=\sigma_{0}{ }^{2}$ against $H_{1}: \sigma^{2} \neq \sigma_{0}{ }^{2}$.
Q3) Attempt any two of the following:
a) State and prove additive property of Gamma distribution.
b) Let X and Y are independent chi square random variables with m and n degrees of freedom respectively, show that $U=X+Y$ and $V=\frac{X}{X+Y}$ are independently distributed.
c) Find $(2 \mathrm{r})^{\text {th }}$ central moment $\left(\mu_{2 r}\right)$ of $t$ distribution with $n$ degrees of freedom.

Q4) Attempt any one of the following.
a) i) Let $X_{1}, X_{2}, \ldots \ldots . X_{10}$ be independent and identically distributed $\mathrm{N}(5,10)$ random variates. Calculate $P\left[\overline{\mathrm{X}} \geq 4, \sum_{i=1}^{n}\left(X_{i}-5\right)^{2} \geq 72.67\right]$.
ii) Explain paired $t$-test along with the assumptions made. Give one illustration in which this test can be used.
b) i) A random sample of 10 boys has mean weight of 63.2 kg with standard deviation of 7 kg . Test whether mean population weight of boys is 60 kg .[Use $\alpha=0.05$ ]
ii) Show that median of $F_{n, n}$ is unity.
iii) Let $t_{12}$ follows Student's t-distribution with 12 degrees of freedom find ' $c$ ' such that $P\left(-c<t_{12}<c\right)=0.8$.


