## Time : 2 Hours]

[Max. Marks : 35
Instructions to the candidates:

1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Use of calculator and statistical tables is allowed.
4) Symbols and abbreviations have their usual meanings.

Q1) Attempt each of the following:
a) Choose the correct alternative in each of the following :
i) If a r.v.x has p.d.f.

$$
f(x)=c ;-2 \leq x \leq 2
$$

$=0$; otherwise
then the value of $c$ is
A) +2
B) $1 / 2$
C) 4
D) $1 / 4$
ii) Let X is a symmetric r.v. with $\mathrm{E}(\mathrm{X})=m$. Then $\mathrm{E}(\mathrm{X}-m)^{3}$ is
A) $m$
B) $m^{3}$
C) 0
D) 1
iii) If $\mathrm{X} \rightarrow \mathrm{N}\left(\mu, \sigma^{2}\right)$ with $\mu_{4}=12$ then the value of $\sigma$ is
A) 4
B) 2
C) $\sqrt{2}$
D) 1
b) State whether the given statement is true or false in each of the following:
i) $\mathrm{E}(c)=0$ where $c$ is any constant.
ii) Normal distribution is mesokurtic

Q2) Attempt any two of the following:
a) Let X is continuous r.v. with p.d.f.

$$
\begin{aligned}
f(x) & =4(1-x)^{3} ; 0<x<1 \\
& =0 \quad ; \text { otherwise }
\end{aligned}
$$

Find :
i) $\mathrm{E}(x)$ and
ii) distribution of $\mathrm{Y}=\frac{\mathrm{X}}{1-\mathrm{X}}$.
b) Obtain point of inflexion of normal probability curve.
c) If $\mathrm{X}_{1}, \mathrm{X}_{2}$ are i.i.d. $\operatorname{Exp}(1)$ then show that $\mathrm{Y}=\min \left(\mathrm{X}_{1}, \mathrm{X}_{2}\right) \rightarrow \operatorname{Exp}(2)$.

Q3) Attempt any two of the following :
a) If X is a $r . v$. taking values $(-a, a)$ has p.d.f. $f(x)$ then find the p.d.f. of $\mathrm{Y}=|\mathrm{X}|$.
b) Obtain mode of $\mathrm{N}\left(\mu, \sigma^{2}\right)$.
c) The joint p.d.f. of a two dimensional continuous r.v. (X,Y) is

$$
\begin{aligned}
f(x, y) & =\frac{8}{9} x y & & ; 1 \leq x \leq y \leq 2 \\
& =0 & & ; \text { otherwise }
\end{aligned}
$$

Find :
i) Marginal distribution of X .
ii) Conditional distribution of Y given $\mathrm{X}=x$.

Q4) Attempt any one of the following:
a) i) The joint p.d.f. of a two dimensional continuous r.v. ( $\mathrm{X}, \mathrm{Y}$ ) is

$$
\begin{aligned}
f(x, y) & =2 ; 0<x<y<1 \\
& =0 ; \text { otherwise }
\end{aligned}
$$

Find $\mathrm{E}(\mathrm{Y} \mid \mathrm{X}=x)$
ii) If $\mathrm{X} \rightarrow \mathrm{U}[a, b]$ then find median of X .
b) i) The p.d.f. of a continuous r.v. X is

$$
\begin{aligned}
f(x) & =\frac{1}{2} ;-1 \leq x \leq 1 \\
& =0 \quad ; \text { otherwise }
\end{aligned}
$$

Find m.g.f. of X. Also find $\mathrm{E}(x)$.
ii) State and prove lack of memory property of exponential distribution.

