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Inclusion exclusion principle for 3 set :-

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

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2M Q.1] Find the cardinality of null set

$\Longrightarrow$ we know that,

null set is denoted by ϕ

$\therefore |\phi| = 0$

5M Q.2] Find the no. of integers between 200 & 500 (both inclusive) which are divisible by 2 or 3 or 7

$\Longrightarrow$ Let $U = \{200, \ldots\ldots 500\}$

$A = \{x \in U \mid x \text{ is divisible by } 2\}$

$B = \{x \in U \mid x \text{ is divisible by } 3\}$

$C = \{x \in U \mid x \text{ is divisible by } 7\}$

$$|A| = \frac{[500]}{2} - \frac{[199]}{2}$$

$= 250 - 99$

$|A| = 151$

$$|B| = \frac{[500]}{3} - \frac{[199]}{3}$$

$= 166 - 66$

$|B| = 100$

$$|C| = \frac{[500]}{7} - \frac{[199]}{7}$$

$= 71 - 28$

$|C| = 43$

$$|A \cap B| = \frac{[500]}{2 \times 3} - \frac{[199]}{2 \times 3}$$
$$= 83 - 33$$
$$|A \cap B| = 50$$

$$|B \cap C| = \frac{[500]}{3 \times 7} - \frac{[199]}{3 \times 7}$$
$$= 23 - 9$$
$$|B \cap C| = 14$$

$$|A \cap C| = \frac{[500]}{2 \times 7} - \frac{[199]}{2 \times 7}$$
$$= 35 - 14$$
$$|A \cap C| = 21$$

$$|A \cap B \cap C| = \frac{[500]}{2 \times 3 \times 7} - \frac{[199]}{2 \times 3 \times 7}$$
$$= 11 - 4$$
$$|A \cap B \cap C| = 7$$

By inclusion exclusion principle.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$= 151 + 100 + 43 - 50 - 14 - 21 + 7$$

$$|A \cup B \cup C| = 216$$

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Q3] A class consist of 5 ~~people~~ girls and 7 boys
i) In how many way a commitee of 5 stud[ents] can be form.
ii) In how many ways can a comittee of 3 girls and 2 boys be form.
iii) In how many ways can a comittee of 5 students having atleast 3 girls be form.

$\Longrightarrow$

~~Total no. of students = 5 girls girls~~

Total no. of students = 5 girls + 7 boys
= 12

i) To select 5 students :-

Total no. of students $= {}^{12}C_5$

$= \dfrac{12!}{5!\,7!}$

$= \dfrac{\cancel{12}^{6} \times 11 \times \cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8}^{2} \times \cancel{7!}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{7!}}$

$= 792$

ii) To select 3 girls and 2 boys.

selection can be done in the following way
we can select 3 girls from 5 girls $= {}^5C_3$.
we can select 2 boys from 7 boys $= {}^7C_2$

$\therefore$ To no. of ways $= {}^5C_3 \times {}^7C_2$

$$= \frac{5!}{3!\,2!} \times \frac{7!}{2!\,5!}$$

$$= \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{2}_1 \times \cancel{3!}} \times \frac{7 \times \cancel{6}^{3} \times \cancel{5!}}{\cancel{2}_1 \times \cancel{5!}}$$

$$= 10 \times 21$$

$$= 210$$

iii) If we want to select atleast 3 girls then we consider cases to form a commitee

case 1: If we select 2 boys from 7 boys & 3 girls from 5 girls

$\therefore$ Total no. of ways $= {}^7C_2 \times {}^5C_3$
$= 210$

case 2: ~~If~~ Total no. of ways to select 1 boys from 7 boys & 4 girls from ~~girls~~. 5 girls.

$\therefore$ Total no. of ways $= {}^7C_1 \times {}^5C_4$

$$= \frac{7!}{1!\,6!} \times \frac{5!}{4!\,1!}$$

$$= \frac{7 \times \cancel{6!}}{\cancel{6!}} \times \frac{5 \times \cancel{4!}}{\cancel{4!}}$$

$$= 35$$

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case 3 : To select 0 boys from 7 boys & 5 girls from 5 girls.

$\therefore$ Total no. of ways $= {}^5C_5 \times {}^7C_0$

$= \frac{5!}{5!\,0!} \times 1$

$= 1$

Required no. of ways $= 210 + 35 + 1$

$= 246.$

2M Q. 4] How many different word can be ~~p~~ form by rearranging a letter of word "MATHEMATICS"

$\Longrightarrow$ In "MATHEMATICS" the letters are

M, A, T, H, E, C, I, S.

Total no. of count.

M = 2 E = 1

A = 2 C = 1

T = 2 S = 1

I = 1 H = 1

The length of word = 11

No. of arrangement $= \frac{11!}{2!\,2!\,2!\,1!\,1!\,1!\,1!\,1!}$

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Solve the recurrance relation $a_r - 5a_{r-1} = 0$ with initial condition $a_1 = 20$

The given recurrance relation

$a_r - 5a_{r-1} = 0$

The characteristic eq^n

put $\alpha = a_r$

$a_{r-1} = \text{const} = 1$

$\alpha - 5 = 0$

$\Rightarrow \alpha = 5$

The homogenous solution is $a_r = A(5)^r$

Initial condition $a_1 = 20$

$\cancel{a_r = A(5)^{20}}$

$a_1 = 20 = A(5)^1$

$= 20 = 5A$

$\Rightarrow A = \frac{20}{5} = 4$

Hence required solution is $a_r = 4 \times 5^r$

6) Solve the following recrrance relation

$a_r - a_{r-1} - 6a_{r-2} = 0$, $a_0 = 5$ & $a_1 = 0$

$\Rightarrow$ consider eq^n.

$a_r - a_{r-1} - 6a_{r-2} = 0.$

The characteristic eq^n is

$1\alpha^2 - 1\alpha - 6\alpha^2 = 0$

$\alpha^2 - \alpha 6 = 0$

$\alpha^2 - 3\alpha + 2\alpha - 6 = 0.$

$\alpha(\alpha - 3) + 2(\alpha - 3) = 0.$

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$$(\alpha+2)(\alpha-3)=0$$

$\Longrightarrow \alpha+2=0$ or $\alpha-3=0$

$\Longrightarrow \alpha=-2$ or $\alpha=3$. are roots.

Let initial condition are

$a_0 = 5$ & $a_1 = 0$

homogenous solution is

$$a_r = A_1(-2)^r + A_2(3)^r \quad \text{———} \quad (*)$$

~~the initial~~

$a_0 = 5$ $a_1 = 0$

from $(*)$

$$a_0 = A_1(-2)^0 + A_2(3)^0$$

$$5 = A_1 + A_2 \quad \text{———} \quad (1)$$

$$a_1 = A_1(-2)^1 + A_2(3)^1$$

$$0 = -2A_1 + 3A_2 \quad \text{———} \quad (2)$$

Multiply eq^n 1 by 2

$$10 = 2A_1 + 2A_2 \quad \text{———} \quad 3$$

add eq^n 2 and 3

$$\begin{array}{rl} & 10 = 2A_1 + 2A_2 \\ + & 0 = -2A_1 + 3A_2 \\ \hline & 10 = 5A_2 \end{array}$$

$$A_2 = \frac{10}{5}$$

$$A_2 = 2$$

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put $A_2 = 2$ in eq^n 1

$5 = A_1 + A_2$

$5 = A_1 + 2$

$5 - 2 = A_1$

$A_1 = 3$

The required sol^n = $a_r = 3(-2)^r + (3)^r$

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