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Subject- Mechanics and Properties of Matter

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Topic- Work and Energy

❖ **Work and Energy:**

The concepts of work and energy are closely tied to the concept of force because an applied force can do work on an object and cause a change in energy. **Energy** is defined as the ability to do work.

❖ **Work:**

When a force acts on an object and the object actually moves in the direction of force, then the work is said to be done by the force.

Work done by the force is equal to the product of the force and the displacement of the object in the direction of force.

If under a constant force F the object displaced through a distance s , then work done by the force $W = F * s = F s \cos \theta$

Where, θ is the smaller angle between F and s .

Work is a scalar quantity, Its SI unit is joule and CGS unit is erg.

$$\therefore 1 \text{ joule} = 10^7 \text{ erg}$$

Its dimensional formula is $[ML^2T^{-2}]$.

Work done by a force is zero, if

(a) body is not displaced actually, i.e., $s = 0$

(b) body is displaced perpendicular to the direction of force, i.e., $\theta = 90^\circ$

Work done by a force is positive if angle between F and s is acute angle.

Work done by a force is negative if angle between F and s is obtuse angle.

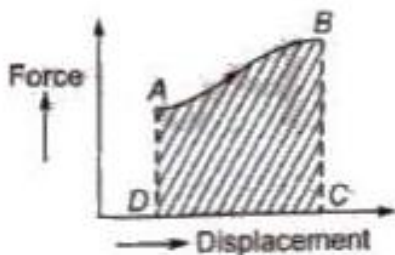
Work done by a constant force depends only on the initial and final Positions and not on the actual path followed between initial and final positions.

Work done in different conditions

(i) Work done by a variable force is given by

$$W = \int F * ds$$

It is equal to the area under the force-displacement graph along with proper sign.



Work done = Area ABCDA

- (ii) Work done in displacing any body under the action of a number of forces is equal to the work done by the resultant force.
- (iii) In equilibrium (static or dynamic), the resultant force is zero therefore resultant work done is zero.
- (iv) If work done by a force during a round trip of a system is zero, then the force is conservative, otherwise it is called non-conservative force.

Gravitational force, electrostatic force, magnetic force, etc are conservative forces. All the central forces are conservative forces.

Frictional force, viscous force, etc are non-conservative forces.

- (v) Work done by the force of gravity on a particle of mass m is given by $W = mgh$ where g is acceleration due to gravity and h is height through which particle is displaced.
- (vi) Work done in compressing or stretching a spring is given by $W = \frac{1}{2} kx^2$ where k is spring constant and x is displacement from mean position.

- (vii) When one end of a spring is attached to a fixed vertical support and a block attached to the free end moves on a horizontal table from $x = x_1$ to $x = x_2$ then

$$W = \frac{1}{2} k (x_2^2 - x_1^2)$$

- (viii) Work done by the couple for an angular displacement θ is given by $W = \tau \cdot \theta$ where τ is the torque of the couple.

❖ Energy:

Energy of a body is its capacity of doing work. It is a scalar quantity. Its SI unit is joule and CGS unit is erg. Its dimensional formula is $[M L^2 T^{-2}]$. There are several types of energies, such as mechanical energy (kinetic energy and potential energy), chemical energy, light energy, heat energy, sound energy, nuclear energy, electric energy etc.

❖ Kinetic Energy:

Kinetic energy is the energy of an object in motion. The expression for kinetic energy can be derived from the definition for work and from kinematic relationships. Consider a force applied parallel to the surface that moves an object with constant acceleration.

The energy possessed by any object by virtue of its motion is called its kinetic energy. Kinetic energy of an object is given by

$$K = \frac{1}{2} mv^2 = \frac{p^2}{2m}$$

where, m = mass of the object, v = velocity of the object and $p = mv$ = momentum of the object.

❖ Potential Energy :

The energy possessed by any object by virtue of its position or configuration is called its potential energy.

Potential energy, also referred to as stored energy, is the ability of a system to do work due to its position or internal structure. Examples are energy stored in a pile

driver at the top of its path or energy stored in a coiled spring. Potential energy is measured in units of joules.

❖ **Gravitational potential energy:**

It is energy of position. First, consider gravitational potential energy near the surface of the earth where the acceleration due to gravity (g) is approximately constant. In this case, an object's gravitational potential energy with respect to some reference level is

$$P.E. = mgh,$$

where h is the vertical distance above the reference level.

To lift an object slowly, a force equal to its weight (mg) is applied through a height (h). The work accomplished is equal to the change in potential energy:

$$W = P.E._f - P.E._o = mgh_f - mgh_o,$$

where the subscripts (f and o) refer to the final and original heights of the body.

Launching a rocket into space requires work to separate the mass of the earth and the rocket to overcome the gravitational force. For large distances from the center of the earth, the above equation is inadequate because g is not constant. The **general form of gravitational potential energy** is

$$P.E. = -GMm/r,$$

where M and m refer to the masses of the two bodies being separated and r is the distance between the centers of the masses. The negative sign is a result of selecting the zero reference at r equal to infinity, that is, at very large **separation**.

❖ **Work-Energy Theorem :**

Work done by a force in displacing a body is equal to change in its kinetic energy.

$$W = \int_{v_1}^{v_2} F \cdot ds = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2 = K_f - K_i = \Delta KE$$

where, K_i = initial kinetic energy and K_f = final kinetic energy.

Regarding the work-energy theorem it is worth noting that

- (i) If W is positive, then $K_f - K_i = \text{positive}$, i.e., $K_f > K_i$ or kinetic energy will increase and vice-versa.
- (ii) This theorem can be applied to non-inertial frames also. In a non-inertial frame it can be written as: Work done by all the forces (including the Pseudo force) = change in kinetic energy in noninertial frame.

❖ Principle of Conservation of Energy:

The sum of all kinds of energies in an isolated system remains constant at all times.

Principle of Conservation of Mechanical Energy:

For conservative forces the sum of kinetic and potential energies of any object remains constant throughout the motion.

According to the quantum physics, mass and energy are not conserved separately but are conserved as a single entity called 'mass-energy'.

Mass-Energy Equivalence:

According to Einstein, the mass can be transformed into energy and vice – versa.

When Δm mass disappears, then produced energy

$$E = \Delta mc^2$$

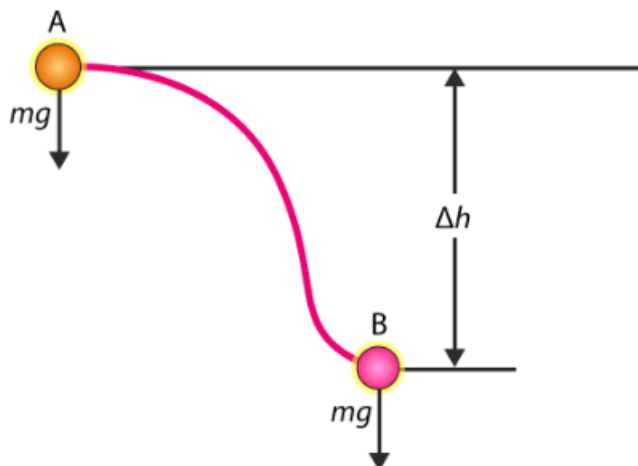
Where, c is the speed of light in vacuum.

❖ Conservative and Non-conservative Force:

A conservative force is a force done in moving a particle from one point to another, such that the force is independent of the path taken by the particle. It depends only on the initial and final position of the particle. Gravitational force and elastic spring forces are two such examples of conservation force.

As the name suggests, conservative force conserves energy. It follows the **law of conservation of energy**. Many forces in nature that we know of like the magnetic force, electrostatic force, gravitational force, etc. are a few **examples of a**

conservative force. Let us understand the concept better with the help of the following example.



Gravitational force acting on a particle

In the given image, the gravitational force acting on the particle has a magnitude equal to mg , where m is the mass of the substance and g is the acceleration due to gravity. The particle moves from point A to point B , and its vertical displacement is given by Δh . The blue curve in the image represents the arbitrary path traveled by the body due to the influence of other forces acting on the body. But the arbitrary path is of no consideration to the force of gravity as it is unaffected by them and therefore can be treated independently. The force of gravity is only dependent on the vertical displacement.

The total work done by gravity on the body is given as follows:

$$W_g = -mg (\Delta h)$$

Where,

- Δh is the difference between the final position (at point B) and the initial position (at point A)
- g is the acceleration due to gravity
- m is the mass of the body

No matter how complicated the path taken by the particle might be, we can easily find out the work done by gravity on the particle using the above expression just by knowing the vertical displacement. From this, we can conclude that the gravitational force doesn't depend on the path taken but only depends on the initial and final position. Hence, the gravitational force is a conservative force.

Properties of Conservative Forces

If a force has the following properties, then it is said to be a conservative force.

- When the force only dependent on the initial and final position irrespective of the path taken.
- In any closed path, the work done by a conservative force is zero.
- The work done by a conservative is reversible.

What is Non-Conservative force?

A non-conservative force is a force for which the work done depends on the path taken. Friction is an example of a non-conservative force. A force is said to be a non-conservative force if it results in the change of mechanical energy, which is nothing but the sum of potential and kinetic energy. The work done by a non-conservative force adds or removes mechanical energy. For example, when work is done by friction, thermal energy is dissipated. The energy lost cannot be fully recovered.

Properties of Non-Conservative Forces:

It has the opposite properties of conservative forces. The properties are given below:

It is path dependent therefore it also depends on the initial and final velocity.

In any closed path, the total work done by a non-conservative force is not zero.

The work done by a non-conservative is irreversible.