[5822]-101

F.Y.B.Sc.

MATHEMATICS

MT-111 : Algebra

(2019 Pattern) (Semester - I) (11111)

Time : 2 Hours]   *[Max. Marks : 35*

Instructions to the candidates:

   1)   *All questions are compulsory.*

   2)   *Figures to the right indicate full marks.*

Q1) Attempt any five of the following.   **[5]**

    a)   Find g.c.d. of 35 and 49.

    b)   Define equivalence relation on z.

    c)   Let $f : R \to R$, $g : R \to R$ defined as $f(x) = x + 1$ and $g(x) = x^2$, find fog.

    d)   If w is a cube root of unity then find the value of $1 + w + w^2$.

    e)   Is $R_1 = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (2, 1), (2, 3)\}$ reflexive relation on the set $A = \{1, 2, 3, 4\}$? Justify?

    f)   Find the value of $\overline{100}$ in Z_3.

    g)   Find the modulus of $z = 1 + \sqrt{3}\hat{z}$.

Q2) A)   Attempt any one of the following.   **[5]**

    i)   State and prove De-Moivre's theorem for an integer indices.

    ii)   Prove that there are n distinct residue classes modulo n in integer.

B)   Attempt any one of the following.   **[5]**

   a)   Find the g.c.d. 'd' of integers 357 and 2210 and express $d = 2210$ $x + 357y$ for some $x, y \in z$.

   b)   Find the remainder of 7^{486} when divided by 13.

Q3) A)   Attempt any one of the following.   **[5]**

   a)   Prove that every partition of non empty set X defines an equivalence relation on X.

   b)   Prove that any two equivalence classes are either identical or disjoint.

   B)   Attempt any one of the following.   **[5]**

   a)   If a, b, c are integers such that $a|bc$ and $(a, b) = 1$ then show that a|c.

   b)   Which elements of z_6 satisfies $x^2 = x$?

Q4) A)   Attempt any one of the following.   **[5]**

   a)   Let $z_1, z_2 \in c$ then prove that $|z_1 + z_2| \le |z_1| + |z_2|$

   b)   State and prove Euclid's lemma.

   B)   Attempt any one of the following.   **[5]**

   a)   Find the expression for $\cos^5\theta$ in terms of cosine of multiple of θ.

   b)   Express $z = 1 + i$ in polar form.

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