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SEAT No. :

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S.Y.B.Sc. (Regular)
MATHEMATICS
MT-231 : Calculus of Several Variables
(2019 Pattern) (Semester - III) (Credit System) (Paper-I) (23111)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*

Q1) Attempt any five of the following. **[5×1=5]**

a) Find the domain and range of $g(x,y)=\sqrt{g-x^2-y^2}$.

b) Where is the function

$$f(x,y)=\frac{x^2-y^2}{x^2+y^2}$$

continuous?

c) Find $\frac{\partial z}{\partial x}$ if $x^3+y^3+z^3+6xyz=1$.

d) Find the critical point of $f(x,y)=x^2+y^2-2x-6y+14$.

e) Write the condition for a critical point (a, b) of a function $f(x, y)$ to be a saddle point.

f) Evaluate

$$\int_0^1\int_0^3 e^{x+3y}\,dx\,dy$$

g) Find the Jacobian of the transformation $x = 5u-v$, $y = u + 3v$.

P.T.O.

Q2) a) Attempt any one of the following. **[5]**

i) If $f(x, y)$ is a function of two variables, write the formulas for $f_x(x,y), f_y(x,y), f_{xx}(x,y), f_{xy}(x,y)$ and $f_{yy}(x,y)$.

ii) State clairaut's theorem. Write laplace's equation in two dimension. Give an example of a function of two variables that satisfies laplace's equation.

b) Attempt any one of the following. **[5]**

i) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

ii) Verify that the function $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$ is a solution of the three dimensional Laplace equation.

Q3) a) Attempt any one of the following. **[5]**

i) If $f(x, y)$ is a homogeneous function of degree n that has continuous second order partial derivatives, then show that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x,y).$$

ii) Explain the second derivative test to classify the critical points of a function of two variables into extreme points and saddle point.

b) Attempt any one of the following. **[5]**

i) If $u = x^4y + y^2z^3$, where $x = rse^t$, $y = rs^2e^{-t}$, and $z = r^2s \sin t$, find the value of $\dfrac{\partial u}{\partial s}$ when $r = 2, s = 1, t = 0$.

ii) A rectangular box without a lid is to be made from $12m^2$ of cardboard. Find the maximum volume of such a box.

Q4) a) Attempt any one of the following. **[5]**

i) State Fubini's theorem. Write the formula for change of cartesian coordinates to polar coordinates in a double integral.

ii) Write the equations of relationship between rectangular coordinates (X, Y, Z) and the spherical coordinates (ρ, θ, ϕ). Hence find the rectangular coordinates of a point $\left(2, \frac{\pi}{2}, \frac{\pi}{2}\right)$ in spherical coordinates.

b) Attempt any one of the following. **[5]**

i) Evaluate

$$\int_{1}^{2}\int_{0}^{2z}\int_{0}^{\ln x} xe^{-y}\, dy\, dx\, dz$$

ii) Evaluate

$$\int_{0}^{1}\int_{3y}^{3} e^{x^2}\, dx\, dy$$

by reversing the order of integration.

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