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P4754

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S.Y. B.Sc.

MATHEMATICS

MT - 231 : Calculus of Several Variables

(23111) (2019 Pattern) (Semester - III) (Paper - I) (Credit System)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any Five of the following : **[5 × 1 = 5]**

a) Let $F(x, y) = 1+\sqrt{4-y^2}$. Evaluate F (3, 1).

b) Let $f(x, y) = x^y$. Find $f_x(x, y)$ and $f_y(x, y)$.

c) Define homogeneous function.

d) Find the critical points of the function $f(x, y) = x^2 + y^2$.

e) Evaluate $\int_1^2 x^2 y \, dy$.

f) Find the Jacobian of the transformation $x = r\cos\theta$, $y = r\sin\theta$.

g) Evaluate $\lim_{(x,y)\to(2,1)} \frac{4-xy}{x^2+3y^2}$.

P.T.O.

Q2) a) Attempt any One of the following : **[5]**

i) State Clairaut's theorem. Define Laplace's equation, harmonic functions and the wave equation.

ii) Define function of three variables, domain, range and level surfaces of function of three variables.

b) Attempt any One of the following : **[5]**

i) Find the limit, if it exists, or show that the limit does not exist :

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz}{x^2+y^2+z^2}.$$

ii) Verify that the function $z = \ln(e^x + e^y)$ is a solution of the differential equation

$$\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0.$$

Q3) a) Attempt any One of the following : **[5]**

i) If $f(x,y)$ is a homogeneous function of degree n, then show that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x,y).$$

ii) Suppose that $z = f(x,y)$ is a differentiable function of x and y, where $x = g(s,t)$ and $y = h(s,t)$ are differentiable functions of s and t. Then prove that

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}.$$

b) Attempt any One of the following : **[5]**

i) Find the local maximum and minimum values and saddle points of the function

$f(x,y) = \sin x \sin y, -\pi < x < \pi, -\pi < y < \pi.$

ii) Use Lagrange multipliers to find the maximum and minimum values of the function

$f(x, y) = x^2 + y^2$

Subject to the constraint $xy = 1$.

Q4) a) Attempt any One of the following : **[5]**

i) Describe the second derivative test to determine the extreme values of functions of two variables. Describe the method of Lagrange multipliers.

ii) Write the relationship between rectangular and polar coordinates. Hence derive the formula for double integration in polar coordinates.

b) Attempt any One of the following : **[5]**

i) Evaluate the double integral

$$\iint_D y^2 dA, D = \{(x, y) \mid -1 \leq y \leq 1, -y-2 \leq x \leq y\}.$$

ii) Evaluate

$$\int_0^1 \int_x^{2x} \int_0^y 2xyz \; dz \; dy \; dx$$

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