Total No. of Questions : 4]

SEAT No. :

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[Total No. of Pages : 3

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F.Y. B.Sc.

MATHEMATICS

MT - 112 : Calculus - I

(CBCS) (2019 Pattern) (Semester - I) (Paper - II) (11112)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*

Q1) Attempt any Five of the following: **[5]**

a) Determine the set $A = \left\{ x \in \mathbb{R} / x < \frac{1}{x}, x > 0 \right\}$.

b) Find first four terms of the sequence (x_n) where $x_n = \frac{1}{n(n+1)}$.

c) Define monotone sequence of real numbers.

d) Find domain of the function $f(x) = \frac{1}{x-3}$.

e) State boundedness theorem for continuous function on an interval.

f) At what points the function $f(x) = \frac{x}{(x-2)(x-4)}$ is discontinuous?

Q2) a) Attempt any one of the following: **[6]**

i) State and prove triangle inequality for real numbers. Hence, prove that $|a-b| \geq ||a|-|b||$, for all $a, b \in \mathbb{R}$.

ii) Define limit of sequence of real numbers and prove that if limit of sequence exists then it is unique.

b) Attempt any one of the following: **[4]**

i) Evaluate : $\lim_{n \to \infty} \frac{2n^2+3}{5n^2+1}$.

ii) Discuss the continuity of the function $f(x)$ at origin,

where $f(x) = \frac{e^{1/x}}{1+e^{1/x}}$; for $x \neq 0$

$= 0$; for $x = 0$

Q3) a) Attempt any one of the following: **[6]**

i) Show that the sequence (x_n) defined by $x_1=1$ and $x_{n+1} = \sqrt{2+x_n}, \forall n \in N$ is monotonic and bounded.

ii) Find the values of α and β, if the function $f(x)$ is continuous in (–3, 3) where, $f(x) = x + \alpha$; $-3 < x < 0$

$= 2x+1$; $0 \leq x > 2$

$= \beta - x$; $2 \leq x < 3$

b) Attempt any one of the following: **[4]**

i) Find the infimum and supremum of the set $S = \left\{1 - \frac{(-1)^n}{n} / n \in N\right\}$.

ii) Let $f : A \to \mathbb{R}$ and c be the cluster point of A. If $\lim_{x \to c} f(x) = L$ and the sequence (x_n) converges to 'c' such that $x_n \neq c$, for all $n \in N$ then prove that $(f(x_n))$ converges to L.

Q4) a) Attempt any one of the following: **[6]**

i) State squeeze theorem and hence show that $\lim_{x \to 0} x \cdot \cos\left(\frac{1}{x}\right) = 0$.

ii) Let $f : \mathrm{I} \to \mathbb{R}$ be continuous on I = [a, b]. If $\mathrm{K} \in \mathbb{R}$ satisfies $f(a) < \mathrm{K} < f(b)$ then prove that there exist a point $c \in \mathrm{I}$ between a and b such that $f(c)$=K.

b) Attempt any <u>one</u> of the following: **[4]**

i) If $\mathrm{A} \subseteq \mathbb{R}$ and $f : \mathrm{A} \to \mathbb{R}$ has limit at $x = c \in \mathbb{R}$, then prove that f is bounded on some neighbourhood of 'c'.

ii) Let $\mathrm{A} \subseteq \mathbb{R}$ and $f, g : \mathrm{A} \to \mathbb{R}$. If f and g are continuous functions at $c \in \mathrm{A}$ then prove that f+g and f·g are also continuous functions at $x = c$.