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F.Y. B.Sc.

MATHEMATICS

MT-122 : Calculus - II

(2019 Pattern) (Semester - II) (Paper - II) (12112)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any Five of the following : **[5]**

a) Whether every continuous function is differentiable? Justify

b) State Lagrange's Mean Value Theorem.

c) Determine whether the differential equation $(2x + y)dx + (y + 2x)dy = 0$ is exact or not.

d) Find the general solution of equation $\frac{dy}{dx} = -\frac{x}{y}$.

e) Find an integrating factor, so that $y\,dx - x\,dy = 0$ is an exact differential equation.

f) Evaluate, $\lim_{x \to 1} \frac{\ln x}{x-1}$.

g) Find n^{th} derivative of the function $y = a^{3x}$, $a > 0$.

Q2) a) Attempt any ONE of the following : **[5]**

i) State and prove Cauchy's Mean Value Theorem.

ii) Show that, n^{th} derivative of $y = e^{ax} \cos(bx + c)$ is $y_n = r^n e^{ax} \cos(bx + c + n\theta)$,

where $r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$.

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b) Attempt any ONE of the following : **[5]**

i) The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , if\ x \neq 0 \\ 0 & , if\ x = 0 \end{cases}$

show that, f is differentiable at $x = 0$.

ii) By using Taylor's series expansion, expand $x^3 + 7x^2 - 6$ in powers of $(x - 3)$.

Q3) a) Attempt any ONE of the following : **[5]**

i) State and prove Leibnitz theorem for n^{th} derivative of the product of two differentiable functions.

ii) If $p(x)$ is continuous on (a, b) then the general solution of the homogeneous equation $\frac{dy}{dx} + p(x)\,y = 0$ on (a, b) is $y = c.\ e^{-Q(x)}$, where $Q(x) = \int p(x)\,dx$, $a < x < b$.

b) Attempt any ONE of the following : **[5]**

i) Evaluate, $\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$.

ii) Solve the differential equation,
$(6xy^2 + 2y)\,dx + (12x^2y + 6x + 3)\,dy = 0$.

Q4) a) Attempt any ONE of the following : **[5]**

i) Explain the method of solving Homogeneous nonlinear equation $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$.

ii) Define exact differential equation. Explain the method of solving exact differential equation.

b) Attempt any ONE of the following : **[5]**

i) Solve the Bernoulli's equation, $x^2\frac{dy}{dx} + 2xy = y^3$.

ii) Verify Rolle's theorem for the function,

$f(x) = 2x^3 + x^2 - 4x - 2$ on $\left[-\sqrt{2}, \sqrt{2}\right]$.

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