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S.Y. B.Sc. (Regular)

MATHEMATICS

MT-241 : LINEAR ALGEBRA

(2019 Pattern) (CBCS) (Semester-IV) (24111)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any five of the following. **[5]**

a) Find the solution set of $7x – 5y = 3$.

b) Let W_1, and W_2 be any two subspaces of vector space V then. Write the condition under which $W_1 \cup W_2$ is a subspace of V.

c) Determine whether the set {(1, 0, 0), (0, 1, 1), (1, 1, 1), (0, –2, 3)} is linearly dependent in $\mathbb{R}^3$.

d) Write standard basis of $M_{2\times2}(\mathbb{R})$, set of all 2×2 matrices with real entries.

e) Define dimension of a vector space.

f) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be multiplication by matrix A. Determine whether T has an inverse where. $A = \begin{bmatrix} 6 & -3 \\ 4 & -2 \end{bmatrix}$

g) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation defined by $T(x, y) = (0, y + 2)$. Determine whether T is linear transformation.

Q2) a) Attempt any one of the following. **[5]**

i) Let $S=\{u_1, u_2....u_r\}$ be set of vectors in $\mathbb{R}^n$. If $r > n$ then prove that set S is linearly dependent.

ii) Let W_1 and W_2 be any two subspaces of vector V then prove that $W_1 \cap W_2$ is subspace of V.

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b) Attempt any one of the following. **[5]**

i) Solve the following system by Gaussian elimination method.

$x + y + 2z = 9$

$2x + 4y - 3z = 1$

$3x + 6y - 5z = 0$

ii) Solve the following system.

$2x + y - 4z + 3w = 0$

$y + 3z - 2w = 0$

$2x + 3y + 2z - w = 0$

$-4x - 3y + 5z - 4w = 0$

Q3) a) Attempt any one of the following. **[5]**

i) Let V be n dimensional vector space and $S = \{v_1, v_2, \ldots v_r\}$ be linearly independent set in V then prove that S can be extended to a basis $S^1 = \{v_1, v_2, \ldots v_r, v_{r+1}, \ldots v_n\}$ of V.

ii) If $A_{m\times n}$ and $B_{n\times p}$ are two matrices then prove that rank

(AB) $\leq$ min {rank (A), rank (B)}.

b) Attempt any one of the following. **[5]**

i) Find a basis and dimension for the solution space of following linear system.

$x + y - z = 0$

$-2x - y + 2z = 0$

$-x + z = 0$

ii) Determine whether the set {(1, 2,–3), (1,–3, 2) (2,–1,5)} is basis of $\mathbb{R}^3$.

Q4) a) Attempt any one of the following. **[5]**

i) Let $T_1 : U \to V$ and $T_2 : V \to W$ be two linear transformations then prove that the composite transformation $T_2 \circ T_1 : U \to W$ is a linear transformation.

ii) Prove that a function $T : V \to W$ is a linear transformation if and only if $T(k_1 u_1 + k_2 u_2) = k_1 T(u_1) + k_2 T(u_2)$, for any vectors u_1 and u_2 in V and scalars k_1 and k_2.

b) Attempt any one of the following. **[5]**

i) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by $T(x, y, z) = (3x + y + z, x - 3y - z)$. Find the matrix of T w.r.t the bases $B=\{(1,1,1), (-1,0,1), (0,0,1)\}$ and $B^1=\{(1, 2) (-1, 1)\}$ of $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively.

ii) Find basis and dimension of range of linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$. Given by $T(x, y, z) = (x + y + 2z, x + z, 2x + y + 3z)$.

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