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SEAT No. :

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S.Y. B.Sc.
MATHEMATICS
MT - 241 : Linear Algebra
(CBCS 2019 Pattern) (Semester - IV) (Regular) (24111)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*

Q1) Attempt any five of the following: **[5]**

a) Find the solution set of the linear equation $5x-4y=7$.

b) Determine whether $W=\{(x,y,z)/x+y+z=1\}$ is a subspace of $\mathbb{R}^3$, justify.

c) Define basis of vector space V.

d) Is the set $W=\{(1,1,3),(2,2,6)\}$ linearly dependent? Justify.

e) Determine a basis for the row space of the matrix $A=\begin{bmatrix}1 & 5\\ 3 & 15\end{bmatrix}$.

f) Use matrix multiplication to find the reflection of (–1, 3) about the x-axis.

g) Define Linear Isomorphism.

Q2) a) Attempt any one of the following: **[5]**

i) If $S=\{v_1,v_2,...,v_n\}$ is a basis for the vector space V, then prove that any vector $v\in V$ can be uniquely expressed as a linear combination of the basis vectors.

ii) Prove that a non-empty subset W of a vector space V is a subspace of V if and only if $\alpha w_1+\beta w_2\in W$, for any scalars α,β and $w_1,w_2\in W$.

P.T.O.

b) Attempt any one of the following: **[5]**

i) Apply Gauss-Jordan method to solve the system of equations

$$x+y+z=9$$
$$2x-3y+4z=13$$
$$3x+4y+5z=40$$

ii) Reduce the matrix

$$A=\begin{bmatrix}1 & 1 & 2\\ 2 & 4 & -3\\ 3 & 6 & -5\end{bmatrix}$$ to row echelon form.

Q3) a) Attempt any one of the following: **[5]**

i) Let V be n-dimensional vector space $(n \geq 1)$, then prove that any linearly independent subset of V with n elements is a basis.

ii) If A and B are two matrices of order $m \times n$, then prove that rank (A+B) $\leq$ rank (A) + rank (B).

b) Attempt any one of the following: **[5]**

i) Find the rank and nullity of the matrix A given by

$$A=\begin{bmatrix}1 & -1 & 3\\ 5 & -4 & -4\\ 7 & -6 & 2\end{bmatrix}$$

ii) Define co-ordinate vector and find the co-ordinate vector of the vector V = (4, 5) relative to the basis $S=\{(2,1),(-1,1)\}$.

Q4) a) Attempt any one of the following: **[5]**

i) Define Kernal of a linear transformation and if $T : V \to W$ is linear transformation, then prove that Kernal of T is a subspace of V.

ii) Let V is a Finite dimensonal vector space and $T : V \to V$ is linear transformation. Prove that T is injective if and only if $\ker(T) = \{0\}$.

b) Attempt any one of the following: **[5]**

i) Find domain, codomain of $T_2 \text{ o } T_1$ and compute $(T_2 \text{ o } T_1)(x, y)$ if $T_1(x, y) = (2x, -3y, x + y)$, $T_2(x, y, z) = (x, -y, x + y)$.

ii) Find the standard matrix for the linear transfrmation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $T(x, y, z) = (3x - 4y + z,\ x + y - z,\ x + 2y + 3z)$.