Sets, Relations and Functions

i) Natural Number : ($\mathbb{N}$)

$\mathbb{N} = \{1, 2, 3, ...\}$

ii) Whole Number :

$W = \{0, 1, 2, 3, ...\}$

iii) Integers :

$I = \mathbb{Z} = \{....., -3, -2, -1, 0, 1, 2, 3, ...\}$

iv) Rational number

$\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0\}$

v) Irrational number.

$\mathbb{Q}^c = \mathbb{R} - \mathbb{Q}$.

Sets :

A set is a collection of objects known a[s] elements or members.
Elements of a set can be any well defin[ed] object, such as numbers, lines, or even se[ts]

- Sets are denoted by capital letters.
  i.e. A, B, C, D, E......

- Elements or objects of set are denoted [by] letters, small letters.
  i.e. a, b, c, d.

If A is an set and x is an element.

i) If x is in A, then we ~~are~~ write $x \in A$, x belong A.

ii) If x is not an element of A then we write $x \notin A$ (x does not belongs to A.

Methods of set.

1. Leasting method (Roaster.
   ex. the natural number between 1 to 10
   $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$

2. Set builder form:
   The natural number between 1 to 10
   $A = \{x / x \in N, 1 < x < 10\}$

3. Empty sets:
   The set that has no element is called empty set or null set and it is denoted by '$\emptyset$' (phi).
   ex. i) The natural number between ~~2 & 3~~ 1 & 2
   $\phi = \{\}$
   ii) The prime numbers between 2 & 3.
   iii) The set of real numbers whose square is negative is an empty set.

Note: A set B said to be known empty if B has atleast one element.
$B = \{5\}$ ex: prime number 3 to 7.

Subset

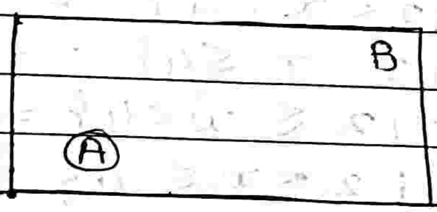
Suppose A, B are two sets we said that A is a subset of B if every element of A is also an element of B.

Notations

$A \subseteq B$ iff $x \in A$ implies $x \in B$.

$A \subseteq B$ iff $x \notin B$ implies $x \notin A$.

- If A is a subset of B then B is called a superset of A and we write it as $B \subseteq A$.



Equality of sets:

Two set A & B are said to be equal if they have the same elements

In other words,

$A = B$ iff $A \subseteq B$ & $B \subseteq A$

Note

1. If $A \subseteq B$ and $A \neq B$ then we say that A is a proper subset of B.

   Notation

   $A \subset B$ or $A \subsetneq B$.

2. Set of natural number is proper subset of $\mathbb{N} \subsetneq W \subsetneq \mathbb{Z} \subsetneq \mathbb{Q} \subsetneq \mathbb{R}$

3. Any set is a subset of itself.

4. The empty set is a subset of any set.

5. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

Intervals

1. (a,b) = open interval
2. $[a,b]$ = closed interval
3. $[a,b)$ = semi closed interval
4. $(a,b]$ = semi open interval

e.g. $(2,4] = \{x \mid 2 < x < 4\} = \{3\}$

$[2,4] = \{x \mid 2 \leq x \leq 4\} = \{2,3,4\}$

$[2,4) = \{x \mid 2 \leq x < 4\} = \{2,3\}$

$(2,4] = \{x \mid 2 < x \leq 4\} = \{3,4\}$.

5. Closed ray :-

$[a,\infty) = \{x \mid a \leq x \text{ i.e. } x \geq a\}$

$(a,\infty) = \{x \mid a < x \text{ i.e. } x > a\}$

Finite and Infinite Set.

A set B is said to be a finite set if it is either empty or consists of finitely many elements, otherwise, the set B is infinite set

e.g. finite set :-

$A = \{1,2,3,4)$

eg. Infinite

$\mathbb{N} = \{1,2,3,\ldots\ldots\}$

$W = \{0,1,2,3,\ldots\ldots\ldots\}$

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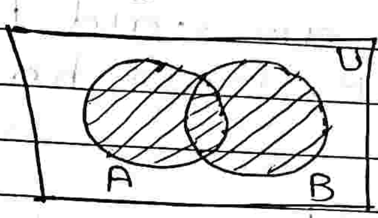
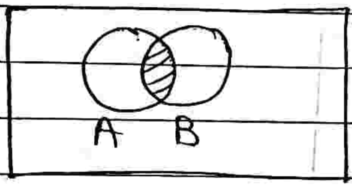
*Operations on sets (∪) :-

1. Union of sets:
let A, B be two sets the union of A & B is the set defined by
$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$
i.e. $x \in A \cup B$ iff $x \in A$ or $x \in B$.
The union of two sets in the set obtained by collecting all the elements of both the set



2. Intersection of set (∩):
Let A, B be the two sets the intersection of A & B is the set defined by
$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$
i.e. $x \in A \cap B$ iff $x \in A$ & $x \in A$



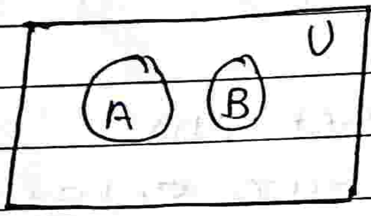
The intersection of two sets is the collection of all elements which are common to both sides.

Note :-
For any two sets A & B
i) $A, B \subseteq A \cup B$
ii $A \cap B \subseteq A, B$

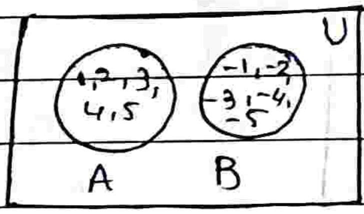
Note 2 :
IF $A \subseteq B$ the $A \cap B = B$ and $A \cap B = D$

e.g. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$ then
Find $A \cup B$, $A \cap B$?
⟹ $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$
$A \cap B = \{1, 2, 3, 4, 5\}$

Disjoint Sets :-
Two set A & B are said to be disjoint if $A \cap B = \phi$ i.e. A & B do not have any element in common.



i) e.g. The set of +ve real number and the set of -ve real number.



Therefore these are disjoint set.
ii) Set of even number and odd numbers are disjoint sets.
iii) $A = \{1, 2, 3, 4, 5\}$ $B = \{6, 7, 8, 9, 10\}$
Therefore $A \cap B = \{\ \} = \phi$
∴ given sets are disjoint sets.

Teacher's Signature: ....

singlon set
e.g. = S = {2} = prime number.

Set difference.
For set A & B the set diff of B from A is defined by
$A \setminus B = \{x \mid x \in A \text{ but } x \notin B\}$
i.e $x \in A \setminus B$ means $x \in A$ & $x \notin A$.
e.g. $A = \{3, 5, 4, 7\}$ $B = \{2, 3, 4, 9\}$
$A \setminus B = \{5, 7\}$ $B \setminus A = \{2, 9\}$

Ex. $A = \{1, 2, 3, 4, 5, 6, 7\}$ $B = \{3, 4, 8, 9\}$ the find
i) $A \cup B$ ii) $A \cap B$ iii) $A \setminus B$ iv) $B \setminus A$ disjoint of A & B
→ i) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
ii) $A \cap B = \{3, 4\}$
iii) $A \setminus B = \{1, 2, 5, 6, 7\}$ iv) $B \setminus A = \{8, 9\}$
v) Disjoint A & B = $\{1, 2, 5, 6, 7, 8, 9\}$

* Symetric difference
The symmetric difference of set A and B is defined as $A \Delta B = \{(A \setminus B) \cup (B \setminus A)$
i.e. $x \in A \Delta B$ if and only if
x belongs to one of A & B, but not both
i.e. $A \Delta B = (A \cup B) \setminus (A \cap B)$

* e.g. $A = \{1, 2, 3, 4, 5\}$
$B = \{2, 3, 4, 7\}$
⇒ $A \setminus B = \{1, 5\}$ OR $A \cup B = \{1, 2, 3, 4, 5, 7\}$
$B \setminus A = \{7\}$ $A \cap B = \{2, 3, 4\}$
$A \Delta B = (A \setminus B) \cup (B \setminus A)$ $A \Delta B = \{A \cup B) \setminus (A \cap B)$
$= \{1, 5, 7\}$ $A \Delta B = \{1, 5, 7\}$

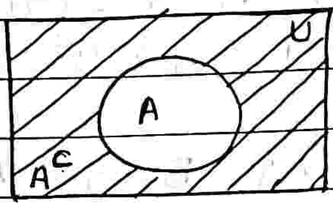
Complement of set :-

If A is a part of universe U, ~~i.e.~~
i.e. $A \subseteq U$

Then the complement of A is written as A^c and defined as

$A^c = \{x \in U : x \notin A\}$

Thus $x \in A^c$ if and only if $x \in U$ & $x \notin A$



e.g. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 4, 6, 8, 10\}$

A^c or $U \setminus A$ or $A' = \{1, 3, 5, 7, 9\}$

De-Morgan's law =

i) $(A \cap B)^c = A^c \cup B^c$

ii) $(A \cup B)^c = A^c \cap B^c$

- Ex. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 3, 4, 7, 9, 10\}$

$B = \{1, 2, 4, 8, 6, 3\}$

$\Rightarrow$ $A \cup B = \{1, 2, 3, 4, 6, 7, 8, 9, 10\}$

$A \cap B = \{2, 3, 4\}$

$A^c = \{1, 5, 6, 8\}$

$B^c = \{5, 7, 9, 10\}$

i) $(A \cap B)^c = A^c \cup B^c$

L.H.S = $\{1, 5, 6, 7, 8, 9, 10\}$

R.H.S = $\{1, 5, 6, 7, 8, 9, 10\}$

L.H.S = R.H.S.

$\therefore$ property (1) hold.

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ii) $(A \cup B)^c = A^c \cap B^c$

L.H.S $= (A \cup B)^c = \{5\}$

R.H.S $= A^c \cap B^c = \{5\}$

L.H.S = R.H.S

property ii hold.

Power set (P)

A set S, The power set of S is defined as family of all subset S. and denotes P(S).

Thus $P(S) = \{A : A \subseteq S\}$

e.g. Let S = a, b

∴ Subset of S = $\{\phi\}, \{a\}, \{b\}, \{a,b\}$

$P(S) = \{\{\phi\}, \{a\}, \{b\}, \{a,b\}\}$

e.g. Let $S = \{a, b, c\}$

subset of S = $\{\phi\}, \{a\}, \{b\}, \{c\}, \{a,b\} \{b,c\} \{a,c\}$ $\{a,b,c\}$

$P(S) = \{\{\phi\}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$

What do you mean by saying

i) $x \notin A \cup B$ ii) $x \notin A \cap B$

i) An element $x \notin A \cup B$ means $x \notin A$ & $x \notin B$

ii) $x \notin A \cap B$ means $x \notin A$ or $x \notin B$

Teacher's Signature:

- $A = \{(x,y) \in R^2, x^2+y^2=1\}$
  $B = \{(x,y) \in R^2, x=1\}$

Find $A \cap B = ?$
Do you understand these geometrically

⟹ Let
order pair x, y belongs to R^2
$(x,y) \in R^2$
suppose $(x,y) \in A \cap B$
$\Leftrightarrow (x,y) \in A$ & $(x,y) \in B$
$\Leftrightarrow x^2+y^2=1$ & $x=1$
$\Leftrightarrow 1+y^2=1$ & $x=1$
$\Leftrightarrow y^2=0$ & $x=1$
$\Leftrightarrow y=0$ & $x=1$
Thus $A \cap B = \{(x,y) \in R^2, x=1 \text{ \& } y=0\}$
$= \{(1,0)\}$

Geometrically
A is a circle centred at origin with radius 1 and B is a line passing through 1.
$A \cap B = \{(1,0)\}$ is a singleton set
i.e. B is tangent line to the circle A.

- Let $A = \{(x,y) \in R^2, x^2+y^2=1\}$ and
  $B = \{(x,y) \in R^2, xy=1\}$
prove that set A and set B are disjoint set

⟹ $(x,y) \in R^2$
suppose $(x,y) \in A \cap B$

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As set A and set B are disjoint which means $A \cap B = \phi$ for this we assume their is a single element which in $A \cap B$.

$\therefore$ suppose $(x, y) \in A \cap B$

$\Leftrightarrow (x, y) \in A$ & $(x, y) \in B$

$\Leftrightarrow x^2 + y^2 = 1$ & $xy = 1$

$\Leftrightarrow \left(\frac{1}{y}\right)^2 + y^2 = 1$ & $xy = 1$

$\Leftrightarrow \frac{1}{y^2} + y^2 = 1$ & $xy = 1$

$\Leftrightarrow \left(y - \frac{1}{y}\right)^2 + 2 = 1$ & $xy = 1$

$\Leftrightarrow \left(y - \frac{1}{y}\right)^2 = -2$ & $xy = 1$

$\because (a-b)^2 = a^2 + b^2 - 2ab$

$\left(y - \frac{1}{y}\right)^2 = y^2 + \left(\frac{1}{y}\right)^2 - 2y\left(\frac{1}{y}\right)$

$\left(y - \frac{1}{y}\right)^2 = \left(y^2 + \frac{1}{y}\right)^2 - 2$

$\left(y - \frac{1}{y}\right)^2 + 2 = y + \frac{1}{y^2}.$

24-07-2023

Cartesian product of set

Cartesian product of two set A and B denoted by $A \times B$ and defined as the set
$A \times B = \{(a, b) ; a \in A, b \in B\}$

Example

Let $A = \{1, 2\}$ $B = \{a, b, c\}$

$A \times B = \{(1, a), (1, b) (1, c), (2, a) (2, b) (2, c)\}$

$B \times A = \{(a, 1) (a, 2), (b, 1), (b, 2), (c, 1), (c, 2), (c, 3)\}$

Note that i) $A \times B \neq B \times A$

ii) If set A has m element and set B has n element then $A \times B$ has $m \times n$

iii) If $A \subseteq X$ & $B \subseteq Y$ then $A \times B \subseteq X \times Y$

iv) Two element (a_1, b_1) & $(a_2, b_2) \in A \times B$ are equal if and only if $a_1 = a_2$ & $b_1 = b_2$

* Relation

Let X and Y be non empty set. A relation R from X to Y is subset of cartesian product of $X \times Y$.

$\therefore R \subseteq X \times Y$

If $R \subseteq X \times Y$ then we say that $(x, y) \in R$ then

Example

$A = \{a, b, c\}$

$B = \{1, 2\}$

$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2) (c, 1), (c, 2)\}$

$B \times A = \{(1, a) (1, b) (1, c), (2, a) (2, b), (2, c)\}$

Teacher's Signature:..................

$R; A \to B = \{(a,1),(a,2),(b,1)(b,2)(c,1)\}$

and CR1 but C R2

Identity relation
Let X be a non empty set
$R = \{(x,x); x \in X\}$ is called identity relation on X.

Example
$A = \{1,2,3\}$
$A \times A = \{(1,1)(1,2),(1,3),(2,1),(2,2)(2,3),(3,1)(3,2),(3,3)\}$
$R = \{(1,1),(2,2),(3,3)\}$
In this R is a identity relation and AXA is called as universal relation on A.

- All R, consider the relation defined by
$R = \{(x,y) \in \mathbb{R} \times \mathbb{R};\ x^2+y^2=1\}$
The unit circle centred at origin $\mathbb{R}^2$.
i) Is 1 related 1.
ii) Find the all reals number y such that 0 is related to y.

⟹ i) as $1^2+1^2=1$
$1+1=1$
$2 \neq 1$

we have $(1,1) \notin R$ i.e. 1 is not related to 1

ii) Let ~~x~~ y be a real no. such 0Ry then

$(0,y) \in R$

$\Leftrightarrow 0^2+y^2=1$

$\Leftrightarrow y^2=1$

$\Leftrightarrow y=\pm 1$

Thus ~~x~~ 0R1 and -1

- Let $X=R$ and let $\mathbb{R}=\{(x,y)\in \mathbb{R}^2, xy=0)$
  When $x \in \mathbb{R}$ related to $y \in \mathbb{R}$

$\Longrightarrow$

~~x∈~~ $xy=0$ i.e. $x=0$ or $y=0$

* Function as a relation

A relation ~~or~~ R from set X to Y is a function iff for each ~~small~~ $x \in X$ their exist unique $y \in Y$ such that order pair $(x,y) \in$ R. In such case we defined $f: X \to Y$ by setting $f(x)=y$ if $(x,y)\in R$.

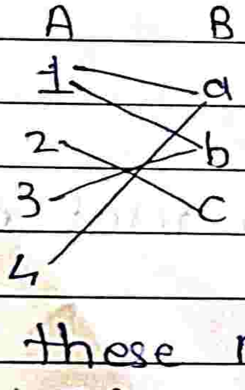
1) Let $A=\{1,2,3,4\}$ & $B=\{a,b,c\}$ Let

i) $R_1=\{(1,a),(1,b),(2,c),(3,b),(4,a)\}$

ii) $R_2=\{(1,c),(3,a),(4,b)\}$

iii) $R_3=\{(1,a),(2,c),(3,c)(4,b)\}$

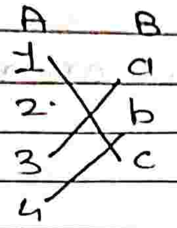
i)



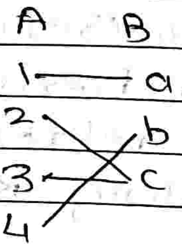
In these relation R_1 is not function from A→B

$\because$ $1 \in A$ is related to two diff element in B.

but we have unique $b \in B$

ii) 

The relation R_2 is not function from $A \to B$ since $2 \in A$ is not related to any element of B.

iii) 

The relation R_3 is a function from $A \to B$ since there is an every element in A has unique image in B.

Types of Relation

- Symmetric Relation
- Reflexive Relation
- Transitive Relation

~~Sy~~ Reflexive Relation

A relation R on a non-empty set X is a said to be reflexive if for all $x \in X$, xRx i.e. each $x \in X$ is related to itself.

Example:
Let R be a relation on natural number defined as xRy iff x is less than y
i.e. $\mathbb{N}$, xRy . $x \leq y$

⟹ as $x \leq x$, for all $x \in \mathbb{N}$
we have xRx, for all $x \in \mathbb{N}$
Hence R is reflexive relation on natural no.

2) Let $A = \{1,2,3,4\}$
i) $R_1 = \{(1,1)(2,2),(3,3),(4,4)\}$
ii) $R_2 = \{(1,1)(1,3)(2,2)(2,4)\}$
iii) $R_3 = \{(1,1)(1,2)(1,3),(1,4),(2,2),(2,4)(3,3)(4,4)\}$
⟹ The relation R_1 and R_3 are reflexive relation on A. but R_2 is not reflexive relation. since (3,3),(4,4) does not belong R_2

3) Let ~~x~~ be a set of lines in plane. if the relation R on X by xRy iff $x \parallel y$
⟹ It is cleared that x is parallel x for each $x \in$ ~~x~~.
∴ R is reflexive relation on X.

3) Let X be a set the subset ~~D = {(x,x)} such that~~. $D = \{(x,x) ; x \in X\}$ is a diagonal as subset of $x \times x$ it is also reflexive index.

II) Symmetric Relation :-
A Relation ~~are~~ R or non-empty set X is said to be symmetric if for all $x,y \in X$, xRy implies yRx.
That is if x related y then y related to x.

Teacher's Signature:..........

ex. 1) Let $A = \{1,2,3,4\}$

i) $R_1 = \{(1,1)(2,2)(3,3)(4,4)\}$

ii) $R_2 = \{(1,1),(1,2),(2,1),(3,4)\}$

$\Longrightarrow$ The relation R_1 is an antisymmetric relation but R_2 is not antisymmetric

$\because (1,2).(2,1) \in R_2$ but $1 \neq 2$

* Inverse Relation

Given relation R from X to Y, the inverse relation R^{-1} of R is the relation from Y to X defined by $R^{-1}\{(y,x) \in Y \times X; (x,y) \in X \times Y\}$

Ex Let $A = \{1,2,3,4\}$ and consider the relation $R = \{(1,2)(1,3)(2,3)(2,4)(3,4)\}$ Find R^{-1}

$\Longrightarrow$ $R^{-1} = \{(2,1)(3,1)(3,2)(4,2)(4,3)\}$

* Transitive Relation

A relation R on a non empty set X is said to be transitive for all $x,y,z \in X$, xRy & yRz implies xRz.
i.e. if x Related to y and y related z then x related to z.

Ex. On $\mathbb{R}$, consider the relation R defined by xRy iff $x<y$ these relation is transitive

$\because x,y,z \in \mathbb{R}$

if $x<y$ & $y<z$ then $x<z$

Suppose $x=4$, $y=7$, $z=11$

here

$x<y$ i.e $4<7$

$y<z$ ie $7<11$

$\Rightarrow x<z$ i.e. $4<11$

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If $x \sim y$

x is equivalance to y

Example.

(4-5 marks)

1 Let us assume that R is a relation on set of integer define by aRb iff a-b is an integer. prove that R is an equivalance relation.

$\implies$ Let

$A = \{ \ldots -2, -1, 0, 1, 2 \ldots \}$ is set of integers.

& aRb iff a-b is an integer

R = a-b is an integer

i.e. $a-b \in A$

1) Reflexive

Let $a \in R$

if $aRa \Rightarrow a-a = 0 \in A$

$\therefore$ R is reflexive relation.

ii) Symmetric

if $aRb \Rightarrow bRa$

Let $aRb = a-b$ is an integer

$\implies a-b \in R$

$-(-a+b) \in R$

$-(b-a) \in R$

$\implies bRa$

R is symmetri relation

iii) Transitive -

$a, b, c \in R$

then $a-b \in R$; a-b is an integer — ①

$b-c \in R$; b-c is an integer — ②

$\implies a-c \in R$; a-c is an integer

$(a-b)+(b-c) \in R$ ...... (addition is closer)

$a-b+b-c \in R$

$a-c \in R$

$\therefore$ R is transitive.

The R is reflexive, symmetric and transitive relation

$\therefore$ It is equivalance relation.

Q2. If R is a relation in the set of integer Z, defined by

$R = \{(x,y) ; x \in Z, y \in Z, x-y$ be divisible by $6\}$

then prove that R is an equivalance relation.

$\rightarrow$ $A = \{ ...., -2, -1, 0, 1, 2 ... \}$ is the set of integer.

$R = \{ (0,6)\ (0,12)\ (6,12) .... \}$

i) Reflexive

original $\forall a \in A, aRa \Longrightarrow (a,a) \in R$

New

$\forall (a,a) \in A, (a,a) R (a,a) \Longrightarrow (a,a) \in R$

Let $(a,a) \in R \Rightarrow a-a$ is divisible by 6

~~$\therefore$ R is~~ 0 is divisible by 6

$\therefore$ R is Reflexive.

ii) Symmetric

original $\forall a,b \in A\ aRb \Longrightarrow bRa$

New - $\forall (a,b) \in A, (a,b) \in R \Longrightarrow (b,a) \in R$

Let

$(a,b) \in R \Longrightarrow a-b$ is divisible by 6

$-(-a+b)$ —11—

$-(b-a)$ —11—

$(b-a)$ —11—

$(b,a) \in R$

$\therefore$ R is symmetric

iii) Transitive

Original - $\forall$ a, b, c $\in$ A, aRb, bRc $\Rightarrow$ aRc
New - $\forall$ (a, b) (b, c) $\in$ A (a, b) $\in$ R (b, c) $\in$ R
Let

(a, b) $\in$ R $\Rightarrow$ a-b is divisible by 6
(b, c) $\in$ R $\Rightarrow$ b-c is divisible by 6.

(a-b) + (b-c) is also divisible by 6
($\because$ closer prop
12 + 6 = 18)

a-b+b-c is divisible by 6,
a-c is divisible by 6.
(a, c) $\in$ R
R is transitive.

- Equivalence Classes

Suppose $\sim$ is an equivalence relation on a non empty set X. For $x \in X$ define $[x] =:= \{y \in X : x \sim y\}$ the subset $[x]$ is called the equivalence classes of x for $\sim$, it is the collection of all those element of X which are equivalent to x.

Example

Let $X = \{1,2,3\}$ then the possible equivalence classes are

$[1] = \{1\}, \{1,2\} \{1,3\} \{1,2,3\}$

$[2] = \{2\}, \{1,2\}, \{2,3\} \{1,2,3\}$

$[3] = \{3\}, \{1,3\}, \{2,3\} \{1,2,3\}$

For Q No. 4 the equivalence classes are

$[a] = \{b \in \mathbb{Z}, a \sim b\} = a-b = nk, k \in \mathbb{Z}$

$= \{b \in \mathbb{Z}, b \sim a\}$ ∵ (symm

$= \{b \in \mathbb{Z}, b-a = nk, k \in \mathbb{Z}\}$

$= \{b \in \mathbb{Z}, b = a+nk, k \in \mathbb{Z}\}$

For Q No. 5 the equivalence classes are

$[x,y] = \{(x_1,y_1) \in \mathbb{R}^2, (x,y) \sim (x,y)\}$

$= \{(x_1,y_1) \in \mathbb{R}^2, x^2+y^2 = x_1^2+y_1^2\}$

$[(3,4)] = \{(x_1,y_1) \in \mathbb{R}^2, 3^2+4^2 = x_1^2+y_1^2\}$

$= \{(x_1,y_1) \in \mathbb{R}^2, 9+16 = x_1^2+y_1^2\}$

$= \{(x_1,y_1) \in \mathbb{R}^2, 25 = x_1^2+y_1^2\}$

$= \{(x_1,y_1) \in \mathbb{R}^2, 5^2 = x_1^2+y_1^2\}$.

$[(3,4)]$ is the circle centred at origin and radius 5

i.e equivalence classes for $\sim$ are the concentric circle centred at origin.

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* Partition of sets -

A partition of a non-empty set X is a pair wise disjoint collection of subset of X whose union is X.

In other words.

A family of subset ~~{Pα ⊆ X α ∈ Λ}~~ $\{P_\alpha \subseteq X \alpha \in \Lambda\}$ of X is a partition of X

i) X is the union of all P_α's

ii) $P_\alpha \cap P_\beta = \phi$, for every $\alpha, \beta \in \Lambda$, $\alpha \neq \beta$.

i) Let ~ be an equivalence relation on a non-empty set X. if $y \in [x]$ then $[y] = [x]$

Prop Thm OR prove that any 2 equivalance classes are either identical or disjoint.

5 marks

Let x be a non-empty set and ~ an equivalence relation on X. Let x and y ∈ X then exactly one of the following is true.

i) $[x] \cap [y] = \phi$

ii) $[x] = [y]$

If i) hold then it is clearly see that 2nd will not prove.

Now we have to show that 1st is ~~false~~ not true then 2nd must be true

$[x] \cap [y] = \phi$ is not true

⇒ There is an at least one element $z \in X$ such that

$z \in [x] \cap [y]$

$\Rightarrow z \in [x]$ & $z \in [y]$ intersection: common.

$\Rightarrow [z] = [x]$ & $[z] = [y]$ .... by previous note. (if $[y] \in [x]$ then $[y] = [x]$, where ~ is an equivalence relation.

$\Rightarrow [x] = [z]$ & $[z] = [y]$

$\Rightarrow [x] = [y]$

i.e. 2nd is true

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$\Rightarrow [x] = [z]$ & $[z] = [y]$ ∵ symmetric relation
$\Rightarrow [x] = [y]$ Transitive relation
that is (ii) is true

Functions

Let X and Y be a non-empty sets. A function (Or mapping or map). f from X to Y is a rule (or correspondance) that assign each element in X, A unique element in Y

Notation
If f from X to Y, $f : X \to Y$
where X is domain
Y is co-domain
If $y = f(x)$ then x is called pre-image of y

$R(f) = \{ y \in Y ; y = f(x) . \text{ for some } x \in X \}$

Example
Let $X = \{1, 2, 3\}$ & $f(x) = 2x + 3$
$f(1) = 2x + 3 = 5$
$f(2) = 2(2) + 3 = 7$
$f(3) = 2(3) + 3 = 9$
$y = \{5, 7, 9\}$
Range of $f(x) = 2x + 3$ is $\{5, 7, 9\}$.

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* Equality of a Function.

Let X, Y, Z, W be a non empty set and $f: X \to Y$ & $g: Z \to W$ be a two function we say that $f = g$ if

i) $X = Z$, $Y = W$ i.e. domain and co-domain

ii) For each $x \in X = Z$

we have $f(x) = g(x)$

Example

Consider the function $f: \mathbb{Z} \to \mathbb{Z}$ &

$g: \mathbb{N} \to \mathbb{Z}$ defined by

$f(x) = g(x) = x^2$ is $f = g$

The given e.g. satisfied the 2nd condition but i.e. for every $x \in \mathbb{Z}$ & $\mathbb{N}$.

$\implies f(x) = g(x)$

if $2 \in \mathbb{N}$ & $-2 \in \mathbb{Z}$

$f(-2) = (-2)^2 = 4$

$g(2) = (2)^2 = 4$

but 1st is not hold. i.e. domain of $f = \mathbb{Z}$

domain of $g = \mathbb{N}$ which is not same.

$\therefore f \neq g$

* One-one function.

A function $f: X \to Y$ is said to be one-one if

for all $x_1, x_2 \in X$, $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$

OR

for all $x_1, x_2 \in X$, ~~$x_1 = x_2$~~ $f(x_1) = f(x_2) \implies x_1 = x_2$.

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Bijection
$F: X \to Y$ is said to be bijective if functio is one-one and onto.
Let F from $\mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x+3$, the F is one one also onto therefore it is bijective.

Composition of function-
$f: x \to Y$
$g: Y \to Z$ be two function the composition of f and g is the function $g \circ f: X \to Z$ defined by $(g \circ f)(x) = g(f(x))$, $\forall x \in X$.

Example
Let f, g is a function : $R \to R$ defined as $f(x) = 2x$ and $g(x) = x^3$ find $g \circ f$
$(g \circ f)(x) = g(f(x))$
$= g(2x)$
$= (2x)^3$
$= 2^3 x^3$
$(g \circ f)(x) = 8x^3$

Q. $f, g : R \to R$
$f(x) = x^2$
$g(x) = 2x+3$
~~a~~ find $g \circ f$ & $f \circ g$. are ~~a~~ they same.
→ $g \circ f(x) = g(f x)$
$= g(x^2)$
$=$ ~~$(2x)^2 + 3$~~ $2x^2+3$
$=$ ~~$4x^2+$~~

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$fog(x) = f(g(x))$

$= f(2x+3)$

$= (2x)^2 + 2(2x)(3) + 3^2$

$= 4x^2 + 12x + 9$

$fog(x) = 4x^2 + 12x + 9$

$\therefore gof \neq fog$

Q 4 marks — Let $g: R \to R$, be a function defined by $g(x) = 5x - 2$. Show that the function g is bijective. Also find formula for g^{-1}.

→ Let $x, y \in R$

suppose

$g(x) = g(y)$

$\Rightarrow 5x - 2 = 5y - 2$

$\Rightarrow 5x - 2 + 2 = 5y$

$\Rightarrow 5x = 5y$

$\Rightarrow x = y$

$\therefore g$ is one-one function.

$x, y \in R$, $g(x) = g(y)$

$y \in R$ such $\exists x \in R$, $g(x) = y$

suppose

For $y \in R$ such that $\exists x \in R$,

$g(x) = y$

$5x - 2 = y$

$5x = y + 2$

$x = \frac{y+2}{5}$

$\Rightarrow$ for any $y \in R$, $\exists x \in R$ such that

$x = \frac{y+2}{5}$

$g(x)$ s.t. $\exists x \in R$

g is onto function.

$\because g$ is one-one & onto function.

$\therefore$ The function (g) is bijective.

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as $g(x)=y$

$\Rightarrow x = g^{-1}(y)$

$\Rightarrow x = \frac{y+2}{5} = g^{-1}(y)$

Q $f, g : R \to R$

$f(x) = x^3+2$

$g(x) = \sqrt[3]{x}$ Find gof & fog.

Let $x \in R$

$gof(x) = g(f(x))$

$= g(x^3+2)$

$gof(x) = \sqrt[3]{x^3+2}$

$fog(x) = f[g(x)]$

$= f(\sqrt[3]{x})$

$= (\sqrt[3]{x})^3 + 2$

$fog(x) = x+2$

$fog \neq gof$

1 marks Let $f, g : R \to R$ defined as $f(x) = x+1$ $g(x) = x^2$

Find fog.

$fog = f[g(x)]$

$= f[x^2]$

$fog = x^2+1$