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S.Y. B.Sc.

MATHEMATICS

MT-242(A) : Vector Calculus

(2019 Pattern) (CBCS) (Semester - IV) (24112A)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any five of the following : **[5]**

a) Evaluate $\lim_{t\to 0}[e^{-t}\cos\overline{i}+e^{-t}\sin\overline{j}+e^{-t}\overline{k}]$.

b) Find the speed along the curve $\overline{r}(t)=(1+2t)\overline{i}+(1+3t)\overline{j}+(4-6t)\overline{k}$ from t = 0 to t = 1.

c) Define flow integral of a vector field along a curve C.

d) Evaluate $\int_C (x+y-z)\,dx$ along the curve $\overline{r}(t)=t\overline{i}-\overline{j}+t^2\overline{k},\ 0\le t\le 1$.

e) State Green's theorem in the plane in normal form.

f) Give parametric representation of the cone $Z=\sqrt{x^2+y^2}, 0\le z\le 1$.

g) Show that the vector field $\overline{F}=(x+y-z)\overline{i}+(2x-y+3z)\overline{j}$ is solenoidal.

Q2) a) Attempt any <u>one</u> of the following : **[5]**

i) If $\overline{r}(t)$ is a differentiable vector function of t and the length of $\overline{r}(t)$ is constant then prove that $\overline{r}\cdot\frac{d\overline{r}}{dt}=0$.

ii) If $\overline{\mathrm{F}}$ is a vector field and C is any closed curve in a region D then prove that the field $\overline{\mathrm{F}}$ is conservative if and only if $\oint_C \overline{\mathrm{F}}\cdot d\overline{r}=0$.

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b) Attempt any one of the following : **[5]**

i) Find the arclength parameter of the curve $\bar{r}(t)=4\cos t\bar{i}+4\sin t\bar{j}+3t\bar{k}$ from the point $t = 0$.

ii) Find the unit tangent vector $\bar{T}$, principal unit normal vector $\bar{N}$ for the plane curve, $\bar{r}(t)=(\cos t+t\sin t)\bar{i}+(\sin t-t\cos t)\bar{j}, t>0$.

Q3) a) Attempt any one of the following : **[5]**

i) State Green's theorem in the plane in tangential form and use it to find counter clock wise circulation for the field $\bar{F}=(x+y)\bar{i}-(x^2+y^2)\bar{j}$ where C is the triangle bounded by $y = 0$, $x = 1$ and $y = x$.

ii) Let C be a smooth curve joining the point A to the point B in the plane and is parametrized by $\bar{r}(t)$. Let f be a differentiable function with a continuous gradient vector $\bar{F}=\nabla f$ on a domain D containing C. Then prove that $\int_C \bar{F}.dr = f(B)-f(A)$.

b) Attempt any one of the following : **[5]**

i) Find the workdone by the force field $\bar{F}=x\bar{i}+3xy\bar{j}-(x+z)\bar{k}$ over the curve $\bar{r}(t)=(1-t)\bar{i}+(4+t)\bar{j}+(2-t)\bar{k}$, $0\le t\le 1$.

ii) Integrate $G(x, y, z) = x^2$ over the cone $z=\sqrt{x^2+y^2}, 0\le z\le 1$.

Q4) a) Attempt any one of the following : **[5]**

i) Define the curl of a vector field $\bar{F}$ and determine whether the field

$$\bar{F}=\left(4y^2+\frac{3x^2y}{z^2}\right)\bar{i}+\left(8xy+\frac{x^3}{z^2}\right)\bar{j}+\left(11-\frac{2x^3y}{z^3}\right)\bar{k}$$ is conservative.

ii) Define surface integral of a scalar function and evaluate $\iint_S 6xy\, dS$ where S is the portion of the plane $x + y + z = 1$ that lies in the first octant and is in the front of the yz - plane.

b) Attempt any <u>one</u> of the following : **[5]**

i) Use the divergence theorem to evaluate $\iint_S \bar{F}.\bar{n}d\sigma$ where $\bar{F} = (3x + z)\bar{i} + (y^2 - \sin xz)\bar{j} + (xz + ye^x)\bar{k}$ and S is the surface of the box $0 \le x \le 1$, $0 \le y \le 3$, $0 \le z \le 2$.

ii) State stoke's theorem and use it to evaluate $\iint_S curl\bar{F}.\bar{n}d\sigma$ where $\bar{F} = xz\bar{i} + yz\bar{j} + xy\bar{k}$, such that S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies in the cylinder $x^2 + y^2 = 1$ above $x - y$ plane.

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