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SEAT No. :

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S.Y.B.Sc.
MATHEMATICS
MT-242 (A): Vector Calculus
(CBCS 2019 Pattern) (Semester-IV) (24112A)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Attempt any Five of the following.***

Q1) Attempt any Five of the following. **[5]**

a) Evaluate $\lim_{t\to 0}[e^{-t}\cos t\,\bar{i} + e^{-t}\sin t\,\bar{j} + e^{-t}\bar{k}]$.

b) Find the unit tangent vector $\bar{T}$ for the vector function $\bar{r}(t) = (t^2+1)\bar{i} + (3-t)\bar{j} + t^3\bar{k}$.

c) Evaluate $\int_C 4x^2 ds$, where C is the circle, $\bar{r}(t) = 2\cos t\,\bar{i} +, 2\sin t\,\bar{j}, 0 \le t \le \frac{\pi}{2}$.

d) Find curl $\bar{F}$, if $\bar{F} = 6x\,\bar{i} + (2y - y^2)\bar{j} + (6z - x^3)\bar{k}$.

e) Find the gradient vector field of the function $f(x,y,z) = xy + yz + xz$.

f) State stoke's Theorem.

g) If S is the colsed surface enclosing a volume V and $\bar{F} = x\bar{i} + 2y\,\bar{j} + 3z\,\bar{k}$, then evaluate $\iint_S \bar{F}\cdot\bar{n}\,ds$

Q2) a) Attempt any one of the following: **[5]**

i) Let $\bar{u}(s) = a(s)\bar{i} + b(s)\bar{j} + c(s)\bar{k}$ be a vector function of ‘s’ and that $s = f(t)$ is a differentiable scalar function of ‘t’ then prove that $\frac{d}{dt}\bar{u}(s) = \frac{d}{dt}f(t)\frac{d}{ds}\bar{u}(f(t))$.

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ii) Define line integral of a vector Field, $\overline{F}$. Evaluate $\int_C \overline{F}\cdot d\,\overline{r}$ where $\overline{F}(x,y,z)=8x^2yz\overline{i}+5z\,\overline{j}-4xy\overline{k}$ and C is the curve given by $\overline{r}(t)=t\overline{i}+t^2\overline{j}+t^3\overline{k}, 0\le t\le 1$.

b) Attempt any one of the following: **[5]**

i) Find the arclength along the curve $\overline{r}(t)=6\sin 2t\overline{i}+6\cos 2t\,\overline{j}+5t\,\overline{k}$, From $t=0$ to $t=\pi$ Also find the unit tangent vector of the curve.

ii) Discuss the continuity of the function

$$\overline{F}(t)=\begin{cases}\left(\dfrac{t^2-1}{\ln t}\right)\overline{i}+\left(\dfrac{\sqrt{t}-1}{l-t}\right)\overline{j}+\tan^{-1}(t)\overline{k}, & if\ t\neq 1\\ 2\overline{i}-\dfrac{1}{2}\overline{j}+\dfrac{\pi}{4}\overline{k}, & if\ t=1\end{cases}$$

Q3) a) Attempt any one of the following: **[5]**

i) If $f(x,y,z)$ is a differentiable Function and (is a continous Path joining points A (x_0,y_0,z_0) and B (x_1,y_1,z_1) then prove that $\int_c \nabla f\cdot d\overline{r}=f(B)-f(A)$

ii) Give Parametrization of a sphere of radius 'a' Find the surface area of sphere $x^2+y^2+z^2=a^2$.

b) Attempt any one of the following: **[5]**

i) Determine whether the vector field,

$\overline{F}=(2x^3y^4+x)\,\overline{i}+(2x^4y^3+4)\,\overline{j}$ is conservative and find a potential Function for $\overline{F}$.

ii) Evaluate $\iint_S y\,d\sigma$ where S is the portion of the cylinder $x^2+y^2=3$ that lies between $z=0$ and $z=6$.

Q4) a) Attempt any one of the following: **[5]**

i) Define curl of a vector Function $\overline{F}$. Check whether $\overline{F} = 6x\overline{i} + (2y - y^2)\ \overline{j} + (6z - x^3)\ \overline{k}$ is conservative vector field.

ii) State Divergence Theorem and using it find the outward Flux of $\overline{F} = (y - x)\overline{i} + (z - y)\ \overline{j} + (y - x)\ \overline{k}$ across the solid sphere $x^2 + y^2 + z^2 \le 4$.

b) Attempt any one of the following: **[5]**

i) Using Greens theorem, evaluate $\oint_C (6y + x)\ dx + (y + 2x)dy$ where C is the circle $(x - 2)^2 + (y - 3)^2 = 4$.

ii) Using stoke's Theorem, evaluate $\iint_S \nabla \times \overline{F} \cdot \overline{n}\ d\sigma$, where $\overline{F} = z^2\ \overline{i} - 3xy\ \overline{j} + x^3 y^3 \overline{k}$ and S is the part of $z = 5 - x^2 - y^2$, above the plane $z = 1$. Assume that S is oriented upwards.